Proofs for

"Firm Expectations and News: Micro v Macro"

by

Benjamin Born, Zeno Enders, Manuel Menkhoff,

Gernot J. Müller, and Knut Niemann

December 2022

For simplicity and without loss of generality, we have assumed $Var(s_t) = Var(q_{l,t})$ in the main text. The following proofs are for the general case $Var(s_t) \neq Var(q_{l,t})$, in which we define $\bar{v} \equiv Var(q_{l,t})/Var(s_t)$. Hence,

$$\bar{\rho}_q^p = \frac{\sigma_q^2}{\sigma_q^2 + \sigma_e^2} = \varpi_q \bar{v}$$

and

$$\rho_q^p = \hat{\varpi}_q \bar{v} = \Upsilon \varpi_q \bar{v} < \varpi_q \bar{v} = \bar{\rho}_q^p.$$

As before, we assume that firms observe the volatility $Var(s_t)$ of the signal and the volatility of idiosyncratic demand $Var(q_{l,t})$.

D Proofs

Proof of Proposition 1 Calculating the expectation error of firms for idiosyncratic output, using demand equation (A-6), the island-specific demand (A-7), and the price-level equation (A-13), yields

$$\begin{aligned} FE_{j,l,t} &= \Delta y_{j,l,t} - E_{j,l,t}\Delta y_{j,l,t} = \gamma \frac{n-1}{n}\left(p_t - E_{j,l,t}p_t\right) + \tilde{y}_{l,t} - E_{j,l,t}\tilde{y}_{l,t} \\ &= \frac{n-1}{n}\left[(\gamma-1)\bar{k}_3 + \delta_x^h(1+\bar{k}_3)\right]\left(\varepsilon_t - E_{j,l,t}\varepsilon_t\right) + q_t - E_{j,l,t}q_t + \sum_{m\in\mathcal{B}_{l,t}} \frac{\bar{q}_{k,t}}{n} \\ &\equiv \Lambda\left(\varepsilon_t - E_{j,l,t}\varepsilon_t\right) + q_t - E_{j,l,t}q_t + \sum_{m\in\mathcal{B}_{l,t}} \frac{\bar{q}_{k,t}}{n}, \end{aligned} \tag{A-16}$$

where the Euler equations (A-5) of customers of island l is used in the second equation. The effect Λ of the expectation error regarding aggregate technology innovations $\varepsilon_t - E_{j,l,t}\varepsilon_t$ on the expectation error regarding own output is negative if

$$\gamma - 1 > -\delta_x^h \frac{1+\bar{k}_3}{\bar{k}_3}. \tag{A-17}$$

Since

$$-\frac{1+\bar{k}_3}{\bar{k}_3} = \frac{(n-1)(1-\alpha)(\gamma-1)(1-\delta_x^p)}{n-\delta_x^h(1-\alpha)[(n-1)\delta_x^p+1]},$$

inequality (A-17) is fulfilled if

$$1 > \delta_x^h(1-\alpha),$$

which is correct, such that $\Lambda < 0$. The gap between expected own and aggregate output can be calculated using (A-6), (A-9), (A-12), and (A-13):

$$\begin{aligned} E_{j,l,t}y_{j,l,t} - E_{j,l,t}y_t &= -\gamma\frac{n-1}{n}\left(p_{j,l,t} - E_{j,l,t}p_t\right) + E_{j,l,t}\tilde{y}_{l,t} - E_{j,l,t}y_t \\ &= \frac{1}{n}\left[-\gamma(n-1)\bar{k}_3 + \delta_x^h(1+\bar{k}_3) - \bar{k}_3\right] E_{j,l,t}\eta_{l,t} \equiv K_1 E_{j,l,t}\eta_{l,t}. \end{aligned} \tag{A-18}$$

Aggregating individual Euler equations (A-3) over all individuals, using (A-13), and (A-14) gives aggregate output as

$$y_t = E_{l,t}x_t + E_{l,t}p_t - p_t - r_t + q_t = x_{t-1} + \underbrace{\left[\delta_x^h - \overline{k}_3(1-\delta_x^h)\right]}_{>0}\varepsilon_t + q_t \underbrace{-\frac{\alpha}{\alpha+\psi(1-\alpha)}}_{<0}\nu_t.$$

Note that, if households have full information ($n \to \infty$), we get $\delta_x^h \to 1$ and $y_t = x_t - \nu_t\alpha/(\alpha+\psi(1-\alpha))$. The signs indicated above result from $0 < -\overline{k}_3 < 1$ (derived above). Forecast revisions are then given by the change in expectations between before and after receiving the private and public signals (that is, between stage one and stage two). The last equation implies

$$E_{j,l,t}y_t - x_{t-1} = \left[\delta_x^h - \overline{k}_3(1-\delta_x^h)\right] E_{j,l,t}\varepsilon_t + \rho_q^p s_t - \frac{\alpha}{\alpha+\psi(1-\alpha)}\nu_t.$$

Using this equation together with equation (A-18) in the forecast revision gives

$$\begin{aligned} FR_{j,l,t} &= E_{j,l,t}(y_{j,l,t} - y_{j,l,t-1}) - E_t(y_{j,l,t} - y_{j,l,t-1}) = E_{j,l,t}y_{j,l,t} - E_{j,l,t}y_t + E_{j,l,t}y_t - E_t y_t \\ &= K_1 E_{j,l,t}\eta_{l,t} + \left[\delta_x^h - \overline{k}_3(1-\delta_x^h)\right] E_{j,l,t}\varepsilon_t + \rho_q^p s_t - \frac{\alpha}{\alpha+\psi(1-\alpha)}\nu_t. \end{aligned}$$

Since

$$E_{j,l,t}\varepsilon_t = \delta_x^p(\varepsilon_t + \eta_{l,t}) \qquad E_{j,l,t}\eta_{l,t} = (1-\delta_x^p)(\varepsilon_t + \eta_{l,t}) \tag{A-19}$$

we can write the above as

$$\begin{aligned} FR_{j,l,t} &= K_1(1-\delta_x^p)(\varepsilon_t + \eta_{l,t}) + \left[\delta_x^h - \bar{k}_3(1-\delta_x^h)\right]\delta_x^p(\varepsilon_t + \eta_{l,t}) + \rho_q^p s_t - \frac{\alpha}{\alpha + \psi(1-\alpha)}\nu_t \\ &\equiv X_1\varepsilon_t + X_1\eta_{l,t} + X_1^q q_t + X_1^q e_t + K_\nu \nu_t. \end{aligned}$$

with

$$X_1 = K_1(1-\delta_x^p) + \left[\delta_x^h - \bar{k}_3(1-\delta_x^h)\right]\delta_x^p \qquad X_1^q = \rho_q^p \qquad K_\nu = -\frac{\alpha}{\alpha + \psi(1-\alpha)}.$$

Similarly, making use of (A-19), the forecast error (A-16) can be written as

$$FE_{j,l,t} = \Lambda\left[(1-\delta_x^p)\varepsilon_t - \delta_x^p\eta_{l,t}\right] + (1-\rho_q^p)q_t - \rho_q^p e_t + \sum_{m\in\mathcal{B}_{l,t}} \frac{\bar{q}_{k,t}}{n}. \tag{A-20}$$

The sign of β of regression (2) can then be determined in two steps. Since both independent variables, forecast revisions and the signal, are correlated, we first regress forecast revisions on the signal, yielding the regression coefficient

$$Coef_1 = \frac{Cov(FR_{j,l,t}, s_t)}{Var(s_t)} = \frac{X_1^q\sigma_q^2 + X_1^q\sigma_e^2}{\sigma_q^2 + \sigma_e^2} = X_1^q.$$

The residual of this regression can therefore be written as $FR_{j,l,t} - Coef_1 s_t$. The sign of the coefficient β of regression (2) then depends on the sign of

$$\begin{aligned} Cov(FE_{j,l,t}; FR_{j,l,t} - Coef_1 s_t) &= Cov(FE_{j,l,t}; FR_{j,l,t}) - Coef_1 Cov(FE_{j,l,t}, s_t) \\ &= \underbrace{(X_1^q - Coef_1)}_{=0} R_e^q + \underbrace{\Lambda X_1}_{<0}\underbrace{R_\eta}_{>0} < 0, \end{aligned}$$

with

$$R_e^q = (1-\rho_q^p)\sigma_q^2 - \rho_q^p\sigma_{e,q}^2 \qquad R_\eta = (1-\delta_x^p)\sigma_\varepsilon^2 - \delta_x^p\sigma_\eta^2.$$

The signs obtain from $\Lambda < 0$ and

$$K_1 = \frac{1}{n}\left[-\gamma(n-1)\bar{k}_3 + \delta_x^h(1+\bar{k}_3) - \bar{k}_3\right] > 0 \qquad X_1 = K_1(1-\delta_x^p) + \left[\delta_x^h - \bar{k}_3(1-\delta_x^h)\right]\delta_x^p > 0,$$

as well as

$$R_\eta > 0 \quad \text{if} \quad \frac{\hat{\sigma}_\eta^2}{\hat{\sigma}_\varepsilon^2} > \frac{\sigma_\eta^2}{\sigma_\varepsilon^2},$$

that is

$$R_\eta > 0 \quad \text{if} \quad \frac{1 - \Upsilon\varpi_a}{\Upsilon\varpi_a} > \frac{1 - \varpi_a}{\varpi_a},$$

which results from the assumption of island illusion, $\Upsilon < 1$. Hence, $\beta < 0$.

The sign of the coefficient δ of regression (2) can equivalently derived by first regressing the forecast revision on the signal, which gives the coefficient

$$Coef_2 = \frac{Cov(FR_{j,l,t}, s_t)}{Var(FR_{j,l,t})} = \frac{X_1^q \sigma_q^2 + X_1^q \sigma_e^2}{X_1^2 \sigma_\varepsilon^2 + X_1^2 \sigma_\eta^2 + (X_1^q)^2 \sigma_q^2 + (X_1^q)^2 \sigma_e^2 + (K_\nu)^2 \sigma_\nu^2},$$

which is positive since $X_1^q > 0$. The sign of δ in regression (2) then depends on the sign of

$$\begin{aligned} Cov(FE_{j,l,t}; s_t - Coef_2(FR_{j,l,t})) &= Cov(FE_{j,l,t}; s_t^q) - Coef_2 Cov(FE_{j,l,t}, FR_{j,l,t}) \\ &= \underbrace{(1 - Coef_2 X_1^q)}_{>0} \underbrace{R_e^q}_{>0} \underbrace{-Coef_2}_{<0} \underbrace{\Lambda X_1}_{<0} R_\eta. \end{aligned}$$

The signs obtain because

$$1 - Coef_2 X_1^q = \frac{X_1^2 \sigma_\varepsilon^2 + X_1^2 \sigma_\eta^2 + (K_\nu)^2 \sigma_\nu^2}{X_1^2 \sigma_\varepsilon^2 + X_1^2 \sigma_\eta^2 + (X_1^q)^2 \sigma_q^2 + (X_1^q)^2 \sigma_e^2 + (K_\nu)^2 \sigma_\nu^2},$$

which is positive but smaller than unity, and

$$R_e^q > 0 \quad \text{if} \quad \frac{\hat{\sigma}_e^2}{\hat{\sigma}_q^2} > \frac{\sigma_e^2}{\sigma_q^2},$$

that is

$$R_e^q > 0 \quad \text{if} \quad \frac{1/\bar{v} - \Upsilon\varpi_q}{\Upsilon\varpi_q} > \frac{1/\bar{v} - \varpi_q}{\varpi_q},$$

which results from the assumption of island illusion. Hence, $\delta > 0$. ■

Proof of Proposition 2

A higher degree of island illusion (a lower Υ) implies...

a) A stronger overreaction to micro news (a lower β) and simultaneously a larger underreaction to the public signal (a larger δ).

The coefficient β of regression (2) is, where results from the proof of Proposition 1 are inserted in the first line

$$\beta = \frac{Cov(FE_{j,l,t}; FR_{j,l,t} - Coef_1 s_t)}{Var(FR_{j,l,t} - Coef_1 s_t)} = \frac{(\overbrace{(X_1^q - Coef_1)}^{=0} R_e^q + \Lambda X_1 R_\eta)}{Var(X_1\varepsilon_t + X_1\eta_{l,t} + X_1^q q_t + X_1^q e_t + K_\nu \nu_t - X_1^q s_t)}$$
$$= \frac{\Lambda[\sigma_\varepsilon^2 - \delta_x^p \sigma_a^2]}{X_1\sigma_a^2 + (K_\nu)^2\sigma_\nu^2/X_1}.$$

First note that the derivative of X_1 with respect to δ_x^p equals

$$\frac{\partial X_1}{\partial \delta_x^p} = \frac{\partial K_1}{\partial \delta_x^p}(1-\delta_x^p) - K_1 + \delta_x^h - \bar{k}_3(1-\delta_x^h) - (1-\delta_x^h)\delta_x^p \frac{\partial \bar{k}_3}{\partial \delta_x^p}.$$

Since

$$\frac{\partial K_1}{\partial \delta_x^p} = \frac{1}{n}\left[-\gamma(n-1) + \delta_x^h - 1\right]\frac{\partial \bar{k}_3}{\partial \delta_x^p}$$

we have

$$\begin{aligned}\frac{\partial X_1}{\partial \delta_x^p} =& - K_1 + \delta_x^h - \bar{k}_3(1-\delta_x^h) + \left\{\frac{1}{n}\left[-\gamma(n-1) + \delta_x^h - 1\right](1-\delta_x^p) - (1-\delta_x^h)\delta_x^p\right\}\frac{\partial \bar{k}_3}{\partial \delta_x^p}\\ =&\bar{k}_3\left[\frac{1}{n}\gamma(n-1) + \frac{1}{n} - (1-\delta_x^h)\right] + \delta_x^h\left[1 - \frac{1}{n}(1+\bar{k}_3)\right] +\\ &\left\{\frac{1}{n}\left[-\gamma(n-1)(1-\delta_x^p) + \delta_x^h - 1\right] + \delta_x^p\frac{1}{n} - \delta_x^p\left[\frac{1}{n}\delta_x^h + 1 - \delta_x^h\right]\right\}\frac{\partial \bar{k}_3}{\partial \delta_x^p}\\ =&\Lambda + \frac{n-1}{n}\left[-\gamma(1-\delta_x^p) + \delta_x^p(\delta_x^h - 1) + (\delta_x^h - 1)/(n-1)\right]\frac{\partial \bar{k}_3}{\partial \delta_x^p}.\end{aligned}$$

Because

$$\frac{\partial \bar{k}_3}{\partial \delta_x^p} = \frac{\delta_x^h}{\Phi - \delta_x^h\Psi}\frac{\partial \Psi}{\partial \delta_x^p} + \frac{n/\Sigma - \delta_x^h\Psi}{(\Phi - \delta_x^h\Psi)^2}\left(\frac{\partial \Phi}{\partial \delta_x^p} - \delta_x^h\frac{\partial \Psi}{\partial \delta_x^p}\right) = \left[\delta_x^h + \bar{k}_3\left((\gamma-1) + \delta_x^h\right)\right]\frac{(n-1)(1-\alpha)}{\Sigma(\Phi - \delta_x^h\Psi)}$$
$$= n\Lambda\frac{1-\alpha}{\Sigma(\Phi - \delta_x^h\Psi)}$$

with

$$\Sigma(\Phi-\delta_x^h\Psi)=(n-1)(1-\alpha)\left[(\gamma-1)(1-\delta_x^p)-\delta_x^p\delta_x^h\right]+n-\delta_x^h(1-\alpha),$$

such that

$$\frac{\partial\bar{k}_3}{\partial\delta_x^p}=\frac{\Lambda}{[-1+\gamma(1-\delta_x^p)+(1-\delta_x^h)\delta_x^p](n-1)/n+1/(1-\alpha)-\delta_x^h/n}<0,$$

we can also write

$$\frac{\partial X_1}{\partial\delta_x^p}=\Lambda\frac{n\alpha/(1-\alpha)}{(n-1)[\gamma(1-\delta_x^p)+(1-\delta_x^h)\delta_x^p]+n\alpha/(1-\alpha)+1-\delta_x^h}\equiv\Lambda K_4<0,$$

with $K_4>0$. The derivative of β with respect to δ_x^p is then positive if

$$\begin{aligned}\frac{\partial\Lambda}{\partial\delta_x^p}R_\eta-\Lambda\sigma_a^2&>\Lambda R_\eta\frac{(\sigma_a^2-K_\nu^2\sigma_\nu^2/X_1^2)}{X_1\sigma_a^2+(K_\nu)^2\sigma_\nu^2/X_1}\frac{\partial X_1}{\partial\delta_x^p}\\ \frac{X_1}{K_4}\frac{K_5R_\eta-\sigma_a^2}{\Lambda R_\eta}&>\frac{\sigma_a^2-K_\nu^2\sigma_\nu^2/X_1^2}{\sigma_a^2+(K_\nu)^2\sigma_\nu^2/X_1^2}<1,\end{aligned}$$

with

$$K_5=\frac{n-1}{n}\frac{\gamma-1+\delta_x^h}{\Lambda}\frac{\partial\bar{k}_3}{\partial\delta_x^p}.$$

The above is fulfilled if

$$\begin{aligned}-\sigma_a^2&<\left(\frac{K_4}{X_1}\Lambda-K_5\right)R_\eta\\ \text{or}\quad-1&<\left(\frac{K_4}{X_1}\Lambda-K_5\right)(\varpi_a-\delta_x^p)\,.\end{aligned}\tag{A-21}$$

Since

$$\frac{K_4}{X_1}\Lambda-K_5=\frac{\frac{\alpha}{1-\alpha}\frac{\Lambda}{X_1}-\frac{n-1}{n}(\gamma-1+\delta_x^p)}{[-1+\gamma(1-\delta_x^p)+(1-\delta_x^h)\delta_x^p](n-1)/n+1/(1-\alpha)-\delta_x^h/n}$$

inequality (A-21) can be written as

$$1-\gamma(1-\delta_x^p)-(1-\delta_x^h)\delta_x^p](n-1)/n-1/(1-\alpha)+\delta_x^h/n<\left[\frac{\alpha}{1-\alpha}\frac{\Lambda}{X_1}-\frac{n-1}{n}(\gamma-1+\delta_x^p)\right](\varpi_a-\delta_x^p)$$

or

$$(\varpi_a-1)(\gamma-1)\frac{n-1}{n}+\frac{\delta_x^p}{n}\left[\varpi_a(n-1)+1\right]-1<\frac{\alpha}{1-\alpha}\left[(\varpi_a-\delta_x^p)\frac{\Lambda}{X_1}+1\right].$$

We start with the left-hand side, which can be expressed as

$$(\varpi_a - 1)(\gamma - 1 + \delta_x^p)\frac{n-1}{n} + \delta_x^P - 1 < 0,$$

where the inequality follows from $\varpi_a, \delta_x^P < 1$. The right-hand side is positive if

$$(\varpi_a - \delta_x^p)\frac{\Lambda}{X_1} + 1 > 0. \tag{A-22}$$

Substituting X_1 and then Λ yields

$$\begin{aligned} \gamma\frac{\overline{k}_3}{\Lambda} &> \frac{1}{n-1} + \varpi_a \\ \gamma &> \frac{n-1}{n}\left[(\gamma-1) + \delta_x^h\left(1 + \frac{1}{\overline{k}_3}\right)\right]\left(\frac{1}{n-1} + \varpi_a\right) \\ \gamma\underbrace{(1-\varpi_a)}_{>0} &> \underbrace{\left[\delta_x^h - 1 + \frac{\delta_x^h}{\overline{k}_3}\right]}_{<0}\underbrace{\left(\frac{1}{n-1} + \varpi_a\right)}_{>0}, \end{aligned}$$

such that inequality (A-21) is fulfilled and hence

$$\frac{\partial\beta}{\partial\Upsilon} = \underbrace{\frac{\partial\beta}{\partial\delta_x^p}}_{>0}\underbrace{\frac{\partial\delta_x^p}{\partial\Upsilon}}_{>0} > 0,$$

demonstrating that a larger degree of 'island illusion' (a lower Υ) leads to a stronger overreaction to micro news (a lower β).

Concerning the effect of Υ on δ,

$$\delta = \frac{(1 - Coef_2 X_1^q)R_e^q - Coef_2\Lambda X_1 R_\eta}{Var(s_t - Coef_2(FR_{j,l,t}))}$$

$$\beta = \frac{\Lambda X_1 R_\eta}{Var(X_1\varepsilon_t + X_1\eta_{l,t} + X_1^q q_t + X_1^q e_t + K_\nu\nu_t - X_1^q s_t)} \equiv \frac{\Lambda X_1 R_\eta}{V_\beta},$$

such that, also substituting X_1^q,

$$\delta = \frac{(1 - Coef_2\rho_q^p)R_e^q - Coef_2\beta V_\beta}{Var(s_t - Coef_2 FR_{j,l,t})}.$$

Since

$$R_e^q = (1-\rho_q^p)\sigma_q^2 - \rho_q^p \sigma_{e,q}^2 = (1-\Upsilon\varpi_q\bar{v})\varpi_q\bar{v}Var(s_t) - \Upsilon\varpi_q\bar{v}(1-\varpi_q\bar{v})Var(s_t) = (1-\Upsilon)\varpi_q\bar{v}Var(s_t).$$

and, see the proof of Proposition 1,

$$Coef_2 = \frac{Cov(FR_{j,l,t}, s_t)}{Var(FR_{j,l,t})} = \frac{X_1^q\sigma_q^2 + X_1^q\sigma_e^2}{X_1^2\sigma_\varepsilon^2 + X_1^2\sigma_\eta^2 + (X_1^q)^2\sigma_q^2 + (X_1^q)^2\sigma_e^2 + (K_\nu)^2\sigma_\nu^2},$$

such that

$$Var(s_t - Coef_2 FR_{j,l,t}) = (1 - Coef_2)^2 Var(s_t) + Coef_2^2 V_\beta = Var(s_t)\frac{V_\beta}{Var(FR_{j,l,t})},$$

as well as

$$1 - Coef_2\rho_q^p = \frac{X_1^2\sigma_a^2 + (K_\nu)^2\sigma_\nu^2}{Var(FR_{j,l,t})} = \frac{V_\beta}{Var(FR_{j,l,t})}$$

we obtain

$$\begin{aligned}\delta &= \frac{\frac{V_\beta}{Var(FR_{j,l,t})}(1-\Upsilon)\varpi_q\bar{v}Var(s_t) - \frac{\rho_q^p Var(s_t)}{Var(FR_{j,l,t})}\beta V_\beta}{Var(s_t)\frac{V_\beta}{Var(FR_{j,l,t})}} \\ &= \varpi_q\bar{v}\left[1 - \Upsilon(1+\beta)\right].\end{aligned}$$

The derivative of δ w.r.t. Υ is therefore

$$\frac{\partial\delta}{\partial\Upsilon} = -\varpi_q\bar{v}\left(1 + \beta + \Upsilon\frac{\partial\beta}{\partial\Upsilon}\right),$$

where $\frac{\partial\beta}{\partial\Upsilon} > 0$ was derived above. Regarding the size of β, note that

$$\begin{aligned}\beta = \frac{\Lambda X_1\sigma_a^2\varpi_a(1-\Upsilon)}{X_1^2\sigma_a^2 + (K_\nu)^2\sigma_\nu^2} &> -1 \\ X_1\sigma_a^2\left[X_1 + \Lambda\varpi_a(1-\Upsilon)\right] &> -(K_\nu)^2\sigma_\nu^2.\end{aligned}$$

Since we have shown that inequality (A-22) holds, we also know that $X_1 + \Lambda\varpi_a(1-\Upsilon) > 0$, such that $\beta > -1$ and

$$\frac{\partial\delta}{\partial\Upsilon} < 0.$$

Hence, a higher degree of island illusion (a lower Υ) leads to a larger underreation to macro news (a higher δ). ∎

(b) Lower expected profits

As usual, the firm's maximization problem states that profits are maximized if the price is a fixed markup over marginal costs. In linearized form

$$p_{j,l,t} = mc_t,$$

where mc_t are marginal costs, given by

$$\begin{aligned} mc_{j,l,t} &= w_t - a_{l,t} + \frac{1-\alpha}{\alpha}(y_{j,l,t} - a_{l,t}) \\ &= w_t + \frac{1-\alpha}{\alpha} y_{j,l,t} - \frac{1}{\alpha} a_{l,t}. \end{aligned}$$

Since the wage w_t and technology $a_{l,t}$ are known at the time when prices are set (and independent of Υ), we have

$$mc_{j,l,t} - E_{j,l,t} mc_{j,l,t} = \frac{1-\alpha}{\alpha}(y_{j,l,t} - E_{j,l,t} y_{j,l,t}) = \frac{1-\alpha}{\alpha} FE_{j,l,t}.$$

The forecast error $FE_{j,l,t}$ is given by equation (A-20). Its expected value is zero and its variance is minimal at $\Upsilon = 1$, see below. Hence, expected profits are also at their maximum at $\Upsilon = 1$. Furthermore, given that the profit function (at the point of approximation) is concave in $P_{j,l,t}$, the larger the distance to the optimal price, the lower realized profits. ■

(c) A larger variance of the firm-specific forecast error

The forecast error $FE_{j,l,t}$ is given by equation (A-20). Its variance results as

$$\begin{aligned} &Var(FE_{j,l,t}) = \\ &\Lambda^2 \sigma_a^2 \left[(1-\delta_x^p)^2 \varpi_a + (\delta_x^p)^2 (1-\varpi_a)\right] + Var(s_t)\left[(1-\rho_q^p)^2 \varpi_q \bar{v} + (\rho_q^p)^2 (1-\varpi_q \bar{v})\right] + \sum_{m \in \mathcal{B}_{l,t}} \frac{\bar{q}_{k,t}}{n} \\ &= \Lambda^2 \sigma_a^2 \varpi_a \left[(1-\Upsilon)^2 \varpi_a + 1 - \varpi_a\right] + Var(s_t) \varpi_q \bar{v} \left[(1-\Upsilon)^2 \varpi_q \bar{v} + 1 - \varpi_q \bar{v}\right] + \sum_{m \in \mathcal{B}_{l,t}} \frac{\bar{q}_{k,t}}{n}, \end{aligned} \tag{A-23}$$

such that

$$\frac{\partial Var(FE_{j,l,t})}{\partial \Upsilon} = -2(1-\Upsilon)\left[\Lambda^2 \sigma_a^2 \varpi_a^2 + Var(s_t)(\varpi_q \bar{v})^2\right].$$

Hence, $Var(FE_{j,l,t})$ is minimal at $\Upsilon = 1$ and rises as $|1-\Upsilon|$ increases. ■

Proof of Proposition 3

As shown in the proof of Proposition 2 *a)*, δ can be written as

$$\delta = \varpi_q \bar{v} \left[1 - \Upsilon(1+\beta)\right],$$

such that

$$\frac{\partial \delta}{\partial \varpi} = \bar{v} \left[1 - \Upsilon(1+\beta)\right] > 0,$$

where we have used the result $\beta > -1$ from the same proof. That is, a higher attachment to the business cycle (a higher ϖ) leads to a larger underreaction to macro news (a larger δ). ■