University of Heidelberg

Department of Economics

Discussion Paper Series | No. 460

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November 2007

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December 5, 2007

Abstract

In the variable supply auction considered here, the seller decides how many costumers with unit demand to serve after observing their bids. Bidders are uncertain about the seller's cost. We experimentally investigate whether a uniform or a discriminatory price auction is better for the seller in this setting. Exactly as predicted by theory, it turns out that the uniform price auction produces substantially higher bids, and consequently yields higher revenues and profits for the seller. Somewhat surprisingly but again predicted by theory, it also yields a higher number of transactions, which makes it the more efficient auction format.

JEL-classification numbers: *D44, C92.*

Key words: auctions, experiment, discriminatory, uniform.

*We thank Jordi Brandts, Jürgen Eichberger, and Kjell Nyborg as well as seminar participants at UCL, the WZB Berlin, and the universities of Cologne and Heidelberg for very helpful comments. Paul Stange and Pablo Calafiore provided excellent research assistance. Financial support by the Deutsche Forschungsgemeinschaft through SFB 504 is gratefully acknowledged.

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1 Introduction

Suppose you want to sell several homogeneous items (concert tickets, photo prints, tie-dyed T–shirts, stocks in an IPO, etc.). You are uncertain about potential buyers' willingness to pay of for those objects. How should you go about selling the objects if each buyer demands only one item? Given the uncertainty about willingness to pay, an auction is an obvious choice. But should you use a uniform price auction or a discriminatory price auction?

We assume that the seller's production cost (or his reserve price) for each object is his private information.[1] In this case it is of advantage to the seller to design the auction as one with *variable supply*, that is, an auction in which the number of items sold is not fixed in advance but may depend on the submitted bids. After observing the bids, the seller chooses the number of items to sell in a profit maximizing way. Of course, bidders take this into account when deciding on their bids. We also assume that all bidders attach the same value to items sold although this fact is not known to the seller.[2]

In the discriminatory auction each bidder has to pay his bid if being served by the seller. Obviously, the seller will do so if and only if the bid is at least as high as the seller's marginal production cost.

In the uniform auction all bidders served by the seller pay the same price, which equals the lowest bid served.[3] Clearly, if all bids are below marginal cost, the seller will not sell any items. However, if several of the bids are above marginal cost, the seller must decide whether to serve a smaller number of buyers and receive a higher price, or serve a larger number of buyers at a lower price.

Damianov and Becker (2007) analyze this situation for the general case with n bidders and general distributions of production cost. They show that

[1] As an example consider a "secret reserve price" on eBay.

[2] This is mainly for simplicity and can be considered a first step to a more general analysis. But it can also be justified in some situations, e.g. when a resale market exists for the items.

[3] In the literature one sometimes finds an alternative version in which the price equals the highest bid not served. However, in practice the latter version does not seem to be in much use, e.g. most treasury auctions use our version. Furthermore, in the current setting with unit demand by bidders, the highest–bid–not–served rule would not make much sense as the seller could never sell all units.

in every symmetric equilibrium, bidders submit higher bids in the uniform than in the discriminatory auction. In the case of two bidders, this result holds even for all rationalizable bids. Consequently, the uniform auction is more profitable for the seller as it generates higher revenues. Somewhat surprisingly, the uniform auction is also more efficient as it generates a higher trade volume.[4] This may seem counterintuitive as in the discriminatory auction all bids above marginal cost are being served whereas in the uniform auction, some bids above marginal cost are rejected by the seller.

In this paper we experimentally test the predictions of Damianov and Becker (2007). Although variable supply auctions are actually quite frequent in reality,[5] to our knowledge, this institutional feature of auctions has so far not been explored experimentally. We consider an auction with two bidders and a uniform distribution of seller's production cost. Section 2 introduces the experimental design. In Section 3 we derive the theoretical predictions for a uniform cost distribution on $[0, 200]$. In the discriminatory auction, there is a strictly dominant strategy of bidding 100. For the uniform price auction, we sharpen the prediction of Damianov and Becker (2007) and narrow down the set of rationalizable strategies to a relatively small interval of $[133, 170]$. In particular, we obtain clear and testable predictions for the difference between the two auctions. The uniform auction should yield significantly higher bids, higher revenues and profits for the seller, and a higher number of transactions.

In Section 4 we present the experimental results, which are remarkably close to the theoretical predictions. As predicted, bids, profits, revenues, and number of transactions are significantly higher in the uniform auction. Subjects seem to learn how to bid in these auctions as the experimental data are even closer to the theoretical predictions in the second half of the experiment. Other factors, like the extent of experience with auctions such as eBay, do not seem to matter for bidding behavior of our subjects. In Section 5 we conclude. Some proofs and the instructions for the experiment

[4] The latter result holds for all weakly convex distribution functions of production cost.

[5] See Loxley and Salant (2004, p. 224), Nyborg et. al. (2002, p. 422), and Busaba et al. (2001) for examples of variable supply auctions for electricity, Treasury bills, and IPOs, respectively.

are collected in an appendix.

2 Experimental design

In a computerized experiment we studied a series of auction markets with variable supply. In each auction two bidders had the opportunity to bid for objects, which had a common redemption value of $v = 200$ cents (= 2 euros). All payoffs were denominated in actual (euro-)cents such that no exchange rate was necessary.

The seller (whose role was played by the computer program) in each round had constant marginal cost c of production for the objects. However, in each round the marginal costs (in cents) were drawn from an i.i.d. uniform distribution on $\{0, 0.1, 0.2, ..., 199.9, 200.0\}$ and the realizations were unknown to bidders at the time of bidding.[6] The seller could choose to sell 0, 1 or 2 objects in each auction and would always choose the profit maximizing option. This rule was known to subjects. Each bidder submitted a bid for one object in each round. Bids could be any amount (in cents) from the set $\{0, 0.1, 0.2, ..., 219.9, 220.0\}$, which makes for an almost continuous strategy set.

The number of auction periods was 20 in all sessions and this was commonly known. At the beginning of the experiment each bidder received an endowment of 400 cents, which made it impossible to lose money in the experiment. After each auction period, bidders were informed about the size of the two bids, how many units were sold (0,1, or 2), what the payoffs for the two bidders were, what the production cost of the seller was, and their cumulative payoff up to that round. Subjects were asked to record all this information on a "record sheet".

There were two treatments.

- In the *discriminatory* pricing treatment, the computer would sell a unit to each bidder who bid above this period's marginal cost and

[6] To improve the pairwise comparison across treatments, the cost realizations were drawn ahead of time and the same set of cost realizations was used for both of our treatments, *uniform* and *discriminatory*.

subjects had to pay their respective bid.

- In the *uniform* pricing treatment, the auction price equaled the lowest bid that was served by the seller. The seller maximized his profits by either selling 0 (if both bids were below marginal cost), one or two units. Clearly, the seller would sell one unit rather than two if $b_1 - c > 2(b_2 - c)$, where $b_1 > b_2$ are the bids and c the marginal cost. If indifferent, the seller would sell to both bidders.

These rules were explained to subjects in detail and with examples. Several test questions were conducted to make sure subjects understood the auction rules. Instructions (see Appendix B) were written on paper and distributed in the beginning of each session. When subjects were familiar with the rules, we started the first round.

The experiments were conducted in June 2007 in the computer lab of the SFB 504, Mannheim. All subjects were recruited via the ORSEE online recruiting system (Greiner, 2004). For the experiment, we used the z–tree software package provided by Fischbacher (2007). In each session 18 subjects participated, constituting three matching groups of six bidders. When entering the lab, subjects were randomly allocated to computer terminals in the lab such that they could not infer with whom they would interact in a group of six. Each group was independent of the others. Matching within groups was done randomly in each round such that no subject could infer who the other bidder was. For both treatments we had six independent matching groups of subjects – making a total of 72 ($= 2 \times 6 \times 6$) subjects who participated in the experiment.

Subjects were paid the sum of their earnings from all auctions. The average payoff was about 12.40 euros. Experiments lasted less than 60 minutes including instruction time.

3 Theoretical predictions

In this section we derive theoretical predictions about bidding behavior in a two–player discriminatory and a uniform price auction, respectively.[7] Note that the seller is not a player in the auction game as he behaves according to a fixed rule (profit maximization) and is thus part of the auction *rules*. In the experiment the seller was played by a computer program and its rules were known to subjects.

Buyers' redemption values are v. The seller's marginal cost is uniformly distributed over the interval $[0, v]$ and bids are from the interval $[0, 1.1v]$.[8] Since equilibria may well differ for a discrete strategy space, we also consider a finite grid, $\Gamma \subset [0, 1.1v]$ with grid size $0.01v$.

The case of the discriminatory auction is a straightforward decision problem for each bidder. Since the seller serves all bids (weakly) above marginal cost and the price a bidder pays equals his bid, optimal strategies are independent of the other player. The expected payoff of bidder i is given by the payoff conditional on winning times the probability of winning, that is $R_i^D(b_1, b_2) = (v - b_i)b_i/v$. The optimal bid is $b_i^D = v/2$. Since this is independent of the other player's behavior, we have

Proposition 1 *In the discriminatory auction bidding $v/2$ is a strictly dominant strategy.*

Consequently, the expected revenues of the seller are v and the expected number of transactions per round is 1.

The case of the uniform price auction is more complex. The question of whether a bidder will be served and the price to be paid depend on both bids b_1 and b_2. If $b_1 < b_2$, then bidder 1 will be served if it is profitable for the seller to sell both units at the price of b_1, which is the case when $2(b_1 - c) \geq b_2 - c$ or, equivalently, $c \leq 2b_1 - b_2$. Hence, bidder 1 will be served with probability $\text{Prob}(c \leq 2b_1 - b_2)$ if $b_2/2 \leq b_1 < b_2$. If $b_1 < b_2/2$, bidder 1 will not be served.

[7] For the general case with n players and general distributions of valuations see Damianov and Becker (2007).

[8] In the experiment $v = 200$ and bids $b_i \in [0, 220]$.

Let us next consider the case $b_2 \leq b_1$. Both bidders are served and pay b_2 if $2(b_2 - c) \geq b_1 - c$, and the probability for this event is $\text{Prob}(c \leq 2b_2 - b_1)$ for $b_1 \leq 2b_2$ and zero otherwise. Only bidder 1 is served and pays b_1 if $2b_2 - b_1 < c < b_1$. This event occurs with probability $\text{Prob}(2b_2 - b_1 < c \leq b_1)$. For $2b_2 < b_1$, the latter probability reduces to $\text{Prob}(c \leq b_1)$. To summarize, the expected payoff of bidder 1 (the payoff of bidder 2 can be determined analogously) is

$R_1^U(b_1, b_2) =$

$$\begin{cases} (v - b_1)\,\text{Prob}(c \leq b_1) & \text{for } 2b_2 < b_1, \\ (v - b_2)\,\text{Prob}(c \leq 2b_2 - b_1) & \\ \quad + (v - b_1)\,\text{Prob}(2b_2 - b_1 < c \leq b_1) & \text{for } b_2 \leq b_1 \leq 2b_2, \\ (v - b_1)\,\text{Prob}(c \leq 2b_1 - b_2) & \text{for } b_2/2 \leq b_1 < b_2, \\ 0 & \text{for } b_1 < b_2/2. \end{cases} \tag{1}$$

Note first that no symmetric, pure–strategy equilibrium exists for the uniform auction. The intuition for why no pair (b, b), in particular not $(v/2, v/2)$, can be an equilibrium, is as follows. Suppose $b_1 = b_2$ and bidder 1 considers increasing his bid marginally to $b_1 + \varepsilon < v$. If $c > b_1 + \varepsilon$, then no trade takes place anyway. If $c < b_2 - \varepsilon$, bidder 1 is equally well off as without the increase: in both cases he receives an item and pays b_2 since the seller finds it profitable to sell two units. If $b_2 < c < b_1 + \varepsilon$, then bidder 1 will obtain the item and receive payoff $v - b_1 - \varepsilon > 0$. Without the increase, bidder 1's payoff would have been 0. This event happens with probability ε. Finally, if $b_2 - \varepsilon < c < b_2$, bidder 1 will receive item but pays ε more than necessary. Again, this event happens with probability ε. Hence, in total there is a gain from the deviation of $v - b_1 - \varepsilon$ with probability ε and a loss of ε with probability ε. For ε small enough, the expected gain exceeds the expected loss and makes the deviation profitable.[9]

Proposition 2 *In the uniform price auction, strategies below* $(2/3)v$ *and above* $0.84v$ *do not survive the iterated elimination of strictly dominated strategies.*

[9] For high bids $b_1 = b_2$, and in particular for $b_2 = v$, decreasing his bid is also profitable for bidder 1.

Proof. See Appendix. ■

Note that in two–player games the set of rationalizable strategies is identical to the set of strategies that survive the iterative elimination of strictly dominated strategies (see e.g. Pearce, 1984). We will therefore use those concepts interchangeably in this paper. In Figure 1 the set of rationalizable strategies for both players is shown as the square between the $\frac{2}{3}v$- and the $0.85v$-lines. Also shown are the best response correspondences (see the Appendix for details). At the intersections of the best response correspondences there are two asymmetric pure strategy Nash equilibria $(b_i, b_{-i}) = (0.77v, 0.69v)$, $i = 1, 2$.

With the help of the numerical algorithm *Gambit* (McKelvey et al., 2007), we were able (for a grid Γ with grid size $0.01v$) to calculate a symmetric mixed strategy equilibrium but also more than a thousand asymmetric mixed strategy equilibria with support in $[0.69v, 0.81v]$. Given this multiplicity of equilibria, it is best to focus on the set of rationalizable strategies, which necessarily contains the support of all Nash equilibria.

Table 1 summarizes the predictions for the two auction formats given the parameters used in the experiment ($v = 200$). The rationalizable bids are given by Proposition 2. The upper and the lower limits for profits, revenues, and number of transactions are obtained by using the lowest, respectively highest, rationalizable bids for both players. For example, when both bidders bid 133.3, then expected revenues are 133.3 times the expected number of transactions, i.e. $133.3 \times 4/3$ =177.7.

The main prediction is clearly that the uniform auction dominates the discriminatory auction from the seller's viewpoint. Since all rationalizable bids are strictly higher in the uniform auction, seller's profits and revenues exceed those in the discriminatory auction. Moreover, the uniform auction is more efficient because the expected number of transactions is higher.

The last result seems counterintuitive since in the discriminatory auction all bids above marginal cost are being served while in the uniform auction, some bids above marginal cost are rejected by the seller. Nevertheless, bids are so much higher in the uniform auction treatment that they result in a

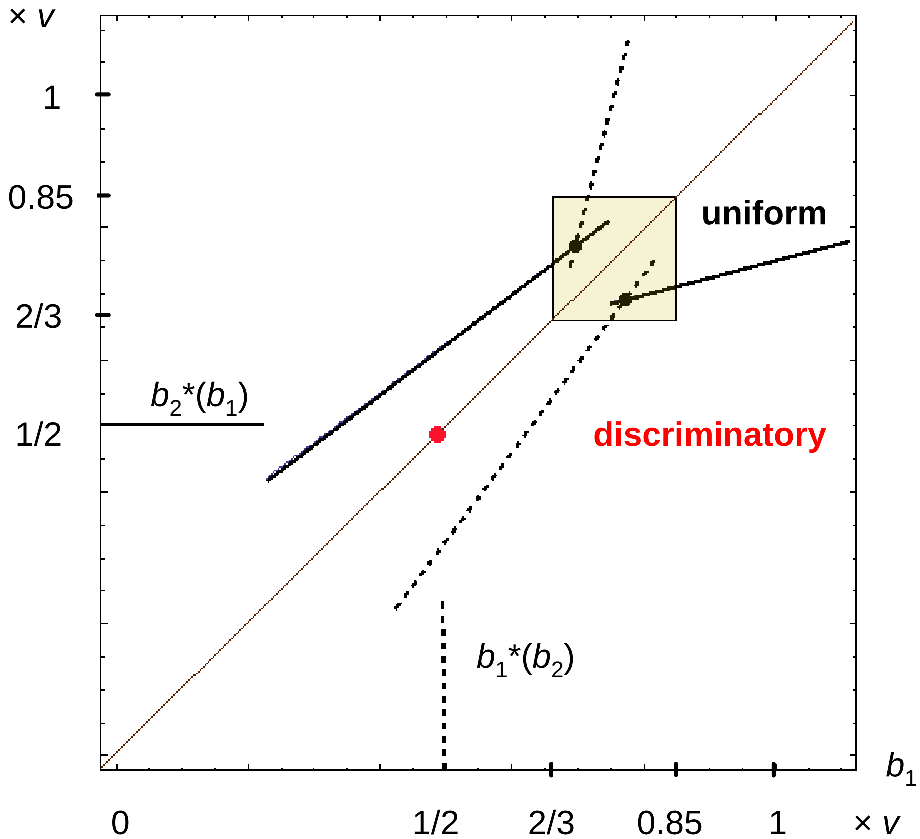

Figure 1: Set of rationalizable bids (shaded area) and pure strategy best reply correspondences for the uniform price auction

Note: The nique equilibrium for discriminatory auction is shown for comparison.

Table 1: **Theoretical predictions for parameters used in the experiment**

Auction	rationalizable bids	expected seller profits	expected seller revenues	expected # of transactions
Uniform	$[133.3, 170]$	$[88.8, 144.5]$	$[177.7, 289.0]$	$[4/3, 1.7]$
Discriminatory	100	50	100	1

Note: All measures are calculated per round and for both bidders.

significantly higher number of transactions.

4 Experimental results

We can now compare the theoretical predictions summarized in Table 1 to the data from our experiment. Table 2 shows mean bids, mean seller profits and revenues, and the average number of transactions per round. The table presents those measures separately for each treatment and data from all rounds and the final 10 rounds, respectively.

Mean values of all four measures are remarkably close to the theoretical predictions for the discriminatory auction. Data for the uniform distribution also lie well within the predicted range. The predictions are even more accurate for the final 10 rounds. As predicted, mean bids are substantially higher in the uniform auction, which results in higher profits and revenues for sellers. Efficiency is also higher in the uniform auction as the average number of transactions per round is more than 1/3 higher for the uniform auction. All differences between the respective measures for our treatments are significantly different at the 1% level of a two–sided MWU test (see, e.g., Siegel and Castellan, 1988) taking each matching group of 6 subjects as an independent observation.

Furthermore, comparing the data to the theoretical predictions for the discriminatory auction (where we have a point prediction) with a Wilcoxon test, we find that the predictions for bids, seller revenues, and number of transactions are not significantly different from the experimental data at

Table 2: **Experimental results**

Treatment	mean bids	mean seller profits	mean seller revenues	mean # of transactions
Uniform (all rounds)	145.10**	97.92**	195.20**	1.35**
Uniform (rounds 11-20)	148.73**	110.51**	199.20**	1.37**
Discr. (all rounds)	105.81**	59.25**	116.39**	1.05**
Discr. (rounds 11-20)	102.78**	64.64**	103.07**	0.98**

Note: ** significantly different from the respective other treatment for the same set of rounds at the 1% level of a two-sided MWU test.

any conventional significance level.

The fact that the data are closer to the predictions for the second half of the experiments suggests that there is learning over time. In fact, Figure 2 shows how bids in the discriminatory auctions converge to the theoretical prediction of 100 while bids in the uniform auction start slightly below the predicted range but increase from there.

Figures 3 and 4 take a closer look at the distribution of individual bids, or pair of bids, respectively. The left panel of Figure 3 shows a histogram of bids in the discriminatory auction. More than 40% of bids lie in the bracket containing the unique Nash equilibrium at 100 (in fact 37.6% are concentrated exactly on bids of 100.0). In contrast, the right panel shows bids in the uniform auction with a mode at 150. Overall, in the discriminatory auction only 11% of bids lie above 133, while in the uniform auction 75% of bids lie above this threshold.

When we consider the pairs of bids that were actually (randomly) matched in the experiment, we get Figure 4.[10] Bids in the discriminatory auction are scattered around 100. Only three pairs of bids are jointly above 133.3. Bids in the uniform auction are predominantly in the area of rationalizable strategies, the square between the lines at 133.3 and 170.

Figure 5 shows for each subject in the uniform price treatment the bid

[10]Note that the matching for the scatter diagram is random and therefore somewhat arbitrary. Nevertheless, these are the outcomes subjects observed in the experiment after each period.

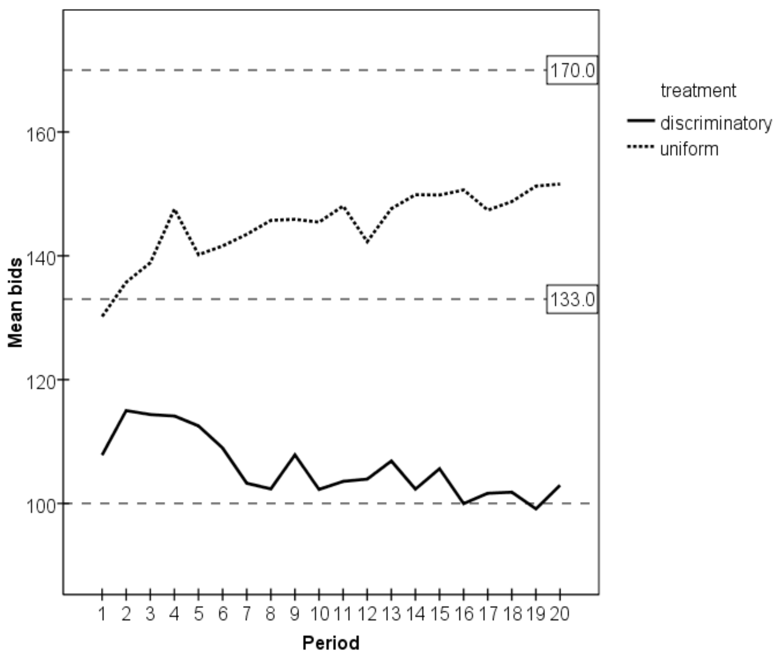

Figure 2: Time path of mean bids in the discriminatory and the uniform auction over the 20 rounds of the experiment.

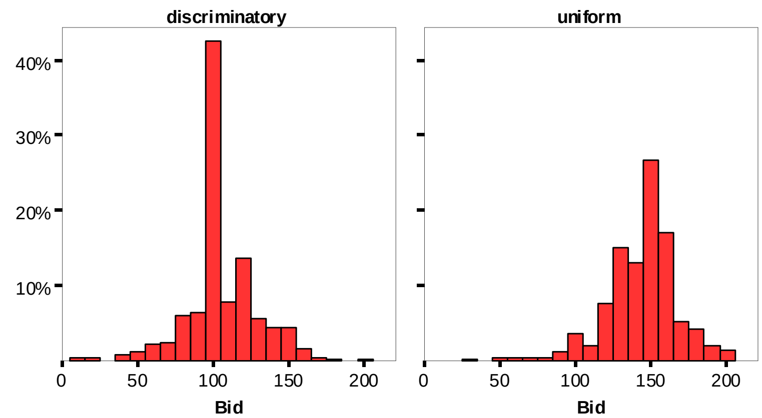

Figure 3: Distribution of individual bids in the discriminatory auction (left panel) and the uniform auction (right panel).

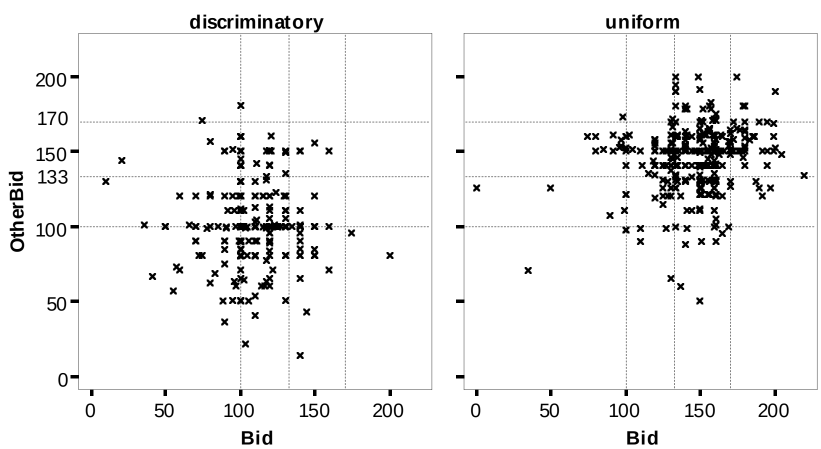

Figure 4: Scatter diagram of pairs of bids (as matched in the experiment) for the discriminatory auction (left panel) and the uniform auction (right panel).

distribution in the second half of the experiment (see Figure 8 in the Appendix for all periods). Given the multiplicity of equilibria, this allows to shed some more light on subjects' strategies. While some subjects seem to play pure strategies, which, in fact, seem fairly close to the asymmetric pure equilibrium strategies, others mix over a broader range of strategies. However, almost all bids lie in the range of rationalizable strategies $[133.3, 170]$.

Finally, we asked subjects in the post–experimental questionnaire how much experience they had with auctions (e.g. eBay). We suspected that subjects with a lot of auction experience might show a bidding behavior that is closer to equilibrium behavior. In fact, a large number of subjects had considerable experience. The average number of auctions in which subjects had participated was 37.1 with a maximum of 300.

However, regressing auction experience together with treatment dummies on bids did not produce any significant coefficients for auction experience. The reason for this result may be that bidding behavior is already so close to equilibrium or rationalizable behavior that experience with auctions cannot make a difference.

5 Related Literature

The uniform and the discriminatory auction formats are widely used in practice for the sale of homogeneous goods. It is therefore not surprising that economists have long debated on the appropriate auction format for a variety of multi–unit markets, e.g. markets for Treasury bills, IPOs, emission permits, and electricity.

Because of the complexity and the specificity of these market environments, the literature does not yield an unambiguous ranking of the two auction formats. Theoretical, empirical, and experimental results vary with the exact institutional structure of the auction under consideration.

The main argument against uniform price auctions is due to Wilson (1979), who presents a model in which bidders announce downward–sloping demand schedules to the seller. By submitting steep demand schedules, bidders can inhibit competition, and thereby sustain equilibria at arbitrarily

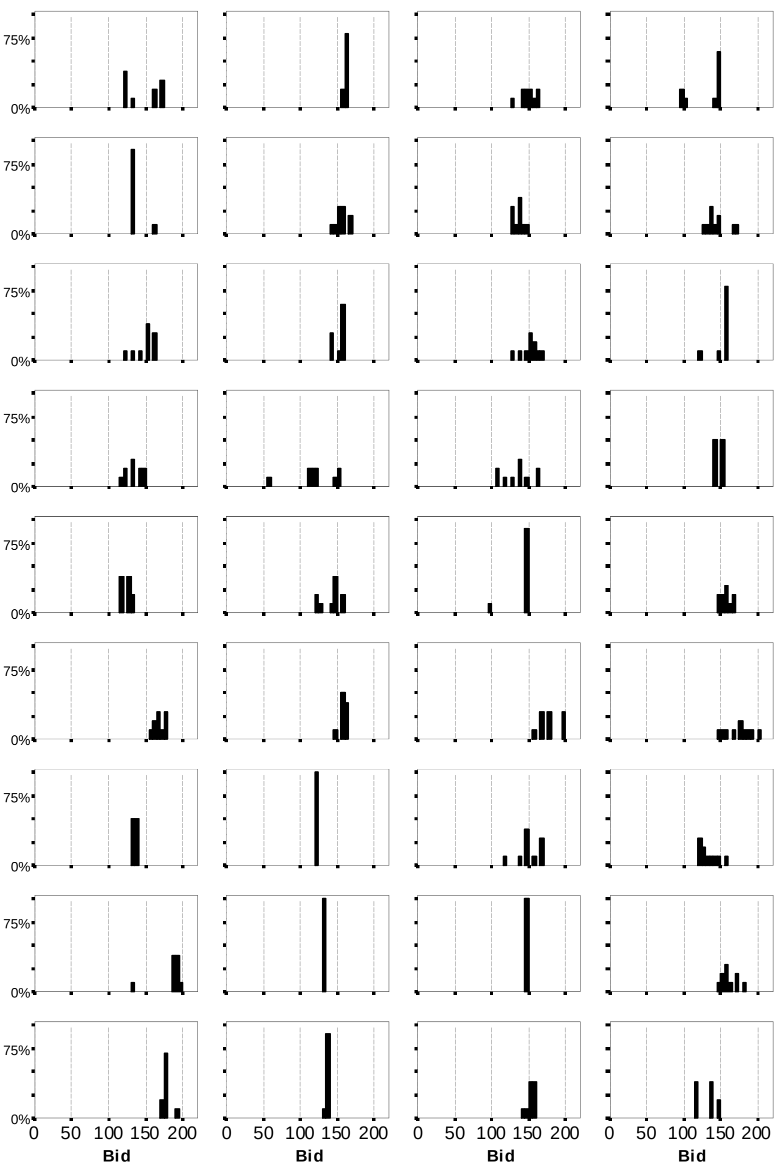

Figure 5: Distribution of bids for each individual subject in the uniform price treatment, periods 11-20.

low prices. This argument has been further developed by Back and Zender (1993) who provide a ranking of the two auctions favoring the discriminatory auction. However, Kremer and Nyborg (2004a, 2004b) show that the underpricing equilibria in the uniform price auction are not robust to alternative formulations of the auction model (e.g. when prices and quantities are discrete or when the seller uses the "pro rata" rationing rule).[11]

The main argument in favor of uniform price auction is based on an application of the "linkage principle" (see Milgrom, 1989, p.16). According to this principle, making the price paid by a bidder dependent on other bids, should increase revenues. However, the principle does not generally apply to multi–unit auctions (see e.g. Perry and Reny, 1999).

The most important strategic feature of the auctions considered in the present paper is that they are a variable supply auctions. The impact of variable supply on bidding has recently been the focus of much theoretical research. Back and Zender (2001) show that the seller can substantially reduce underpricing in the uniform price auction if he retains the right to reduce supply after the bidding. McAdams (2007) demonstrates further that low–price equilibria will be completely eliminated if the seller can both extend and reduce supply quantity. Both models assume that buyers have the same deterministic valuation. In contrast, Lengwiler (1999) introduces uncertainty on the part of the bidders with respect to the marginal cost of the seller. To keep the setting tractable, he allows bidders to submit quantity bids only at two exogenously given price levels, and finds an ambiguous ranking of the uniform price and the discriminatory auction in terms of revenues. For the current paper most relevant are the results of Damianov and Becker (2007), who show that in a setting with variable supply and single–unit demand of bidders, the uniform auctions dominates the discriminatory auction in terms of revenues and profits for the seller and efficiency.

Empirical research, especially on Treasury auctions, is largely in favor of the uniform auction format. In particular, there seems to be little evidence of low–price equilibria in uniform price auctions. In fact, empirical

[11] See also Keloharju et al. (2005) who do not find any evidence for underpricing equilibria in Finnish treasury securities auctions.

evidence suggests that underpricing, as measured by the difference between the auction's stop–out prices and secondary market prices, is higher in discriminatory auctions (see Umlauf, 1993, Nyborg, and Sundaresan, 1996, and Goldreich, 2006). It has also been observed (see e.g. Keloharju et al., 2005, for an overview of this literature) that in many of the auctions for Treasury bonds and IPOs, sellers adjust supply after observing bids, and this effectively changes the strategic structure of the auction game.

Given the lack of definite conclusions from the theoretical literature, a lot of attention has naturally been focused on the use of experimental methods for comparison of the two auctions and the evaluation of policy proposals. By and large, the experimental results either find no clear difference between the two auction formats or favor the uniform price auction.

Smith (1967) compares the uniform price and the discriminatory auctions in a Treasury market experiment in which the quantity to be sold is known to bidders prior to the bidding and the asset has a common resale value. He observes that the variance of bids in the uniform price auction is generally higher but finds no clear ranking in terms of revenues. Goswati, Noe, and Rebello (1996) present the only experiment which favors the discriminatory auction due to the fact that communication among bidders prior to the auction facilitates collusion more in the uniform price auction than in the discriminatory auction. Cox, Smith, and Walker (1985) conduct an experiment in which each buyer demands a single unit and found no significant differences in the revenue generation properties of the two auctions. Miller and Plott (1985) allow for multi–unit demand and fixed supply and observe that the uniform price auction generates higher revenues when demand is elastic and the discriminatory auction performs better when demand is inelastic. Finally, Abbink et al. (2006) compare the uniform, the discriminatory, and an auction format used by the Bank of Spain for selling government bonds. They find that the uniform and the Spanish auction formats outperform the discriminatory auction in terms of revenues for the seller.

Rassenti, Smith, and Wilson (2003) perform an electricity market experiment in which subjects are sellers and submit step supply functions. Demand is stochastic and determined by the computer. Thus, trading quantity in this experiment is endogenous, although not of the type studied in the present experiment. Yet, similarly to the present experiment, the uniform auction is found to be more favorable to the auctioneer.

In a related electricity market experiment, Abbink, Brandts, and McDaniel (2003) focus on the aspect of asymmetric information regarding market demand and find that the discriminatory auction leads to a less efficient outcome. In symmetric information treatments, prices and volatility are not significantly different.

Auctions have also been considered in the design of markets for allocating resources to protect the environment. Cason and Plott (1996) experimentally compare the uniform price double auction to the auction proposed by the Environmental Protection Agency (EPA) in 1990. In the EPA auction, bids are arranged from highest to lowest, and asks are arranged from lowest to highest. Then the lowest ask is matched with the highest bid and trade occurs at the bid price. Thus, from bidders' perspective the EPA auction is similar to the discriminatory auction because bidders are required to pay their bid prices. Similarly to what we observe in the present experiment, discriminatory pricing leads to higher underbidding and lower trade volume and efficiency.

Cason and Gangadharan (2005) perform a procurement experiment in which sellers offer environmental projects and compete in the dimensions of ask price and level of environmental improvement. The regulator spends a certain monetary budget to purchase at most one project from each seller with the goal of achieving the highest environmental benefit. In the uniform price treatment of this experiment, offers are not substantially different from costs,[12] whereas in the discriminatory treatment offers exceed costs substantially.

[12] In their uniform price auction treatment the price is set by the first rejected bid rather than the last accepted bid as we assume in the present experiment.

6 Conclusion

In this paper, we report results of an experiment that compares uniform price auctions to discriminatory auctions in a setting with variable supply. The experimental results are remarkably close to the theoretical predictions. Just as theory predicts, bids are substantially higher in the uniform auction, and, consequently, so are revenues and profits for the seller. Despite the fact that in the discriminatory auction the seller never rejects bids that are above his marginal cost, the number of transactions are higher in the uniform auction, which implies that the uniform auction is also the more efficient auction in this setting.

The theoretical results hold under somewhat restrictive assumptions. It would be desirable to relax some of those assumptions in future work, in particular, the assumption that both bidders have the same value v. We see this assumption mainly as a technical simplification as does much of the literature (see Back and Zender, 2001; McAdams, 2007). We are relatively confident that the results are robust to the introduction of some asymmetry with respect to bidders' values but future research needs to verify this conjecture. We would also conjecture that the experimental results are robust to moderate asymmetry in bidders' values.

Another interesting extension would be to allow for multi–unit demand by bidders. While some progress has been made in the literature,[13] this problems remains a challenge for future research.

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[13] See e.g. Ausubel and Crampton (2002) and Wang and Zender (2002) for the theoretical analysis and List and Lucking-Reiley (2000) and Grimm and Engelmann (2005) for the experimental evidence. However, as far as we know, no one has studied auctions with multi–unit demand and variable supply in an experiment.

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Appendix A: Proofs

Iterative elimination of strictly dominated strategies

Proof of Proposition 2. In order to simplify notation in this Appendix, let $v = 1$ without loss of generality. Applying the uniform distribution of costs, the payoff function (1) becomes

$$R_1^U(b_1, b_2) = \begin{cases} (1-b_1)\min\{b_1, 1\} & \text{for } 2b_2 \le b_1, \\ (1-b_2)\min(2b_2 - b_1, 1) & \\ \quad + (1-b_1)\min\{2b_1 - 2b_2, 1\} & \text{for } b_2 \le b_1 < 2b_2, \\ (1-b_1)\min\{2b_1 - b_2, 1\} & \text{for } b_2/2 \le b_1 < b_2, \\ 0 & \text{for } b_1 < b_2/2. \end{cases} \tag{2}$$

Observe first that bidding above one's valuation of 1 is strictly dominated by bidding 1 since for all points where the partial derivative is defined

$$\frac{\partial R_1^U(b_1, b_2)}{\partial b_1} < 0, \quad \forall b_1 \ge 1 \quad \text{and} \quad \forall b_2 \in [0, 1.1].$$

The intuition for this is obvious. If $b_1 > 1$ and the price bidder 1 pays equals b_1, he is clearly better off lowering his bid. Alternatively, the price bidder 1 pays is determined by b_2, which happens with probability $2b_2 - b_1$. Lowering b_1 increases this probability and makes bidder 1 better off. In the following we shall therefore work with the restricted strategy set $[0, 1]$. The min operators in (2) can then be replaced by their first argument.

Second, we notice that for $b_2 \in [0, 2/3]$ and $b_1 \in \left(b_2/2, \min\{b_2 + \frac{1}{12}, 2/3\}\right)$ (see areas A and B in Figure 6) the expected payoff $R_1^U(b_1, b_2)$ is strictly increasing in b_1; and for $b_1 < b_2/2$ we have $R_1^U(b_1, b_2) = 0$ (see area C in Figure 6).

Third, we observe that pure strategies from the interval $[0, \frac{1}{12})$ are not best responses to any (mixed) strategy of bidder 2.[14] Indeed, if bidder 2 plays a mixed strategy which places a positive probability on the interval $[0, \frac{2}{12})$, then bidding 1/12 will lead to a strictly higher payoff for bidder 1 than using any of the pure strategies from the interval $[0, \frac{1}{12})$. On the other

[14] In two-player games the property of "never a best response" is equivalent to being strictly dominated.

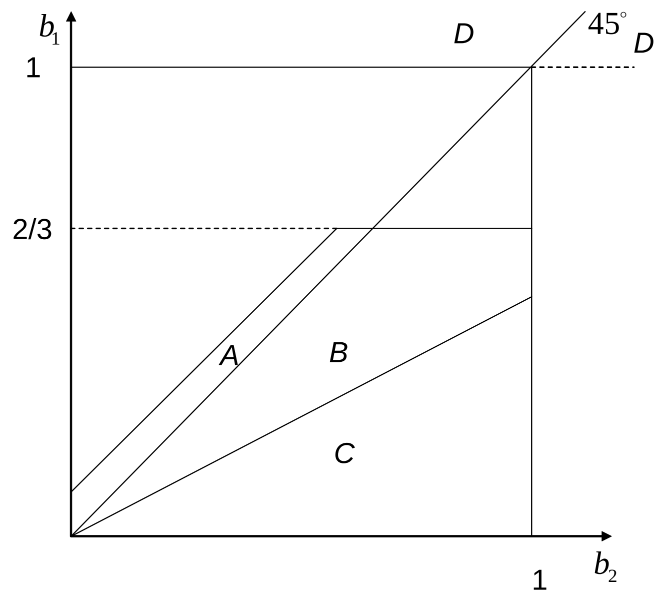

Figure 6: The expected payoff of bidder 1 in the uniform price auction is zero in area C, strictly increasing in b_1 in areas A and B and strictly decreasing in area D.

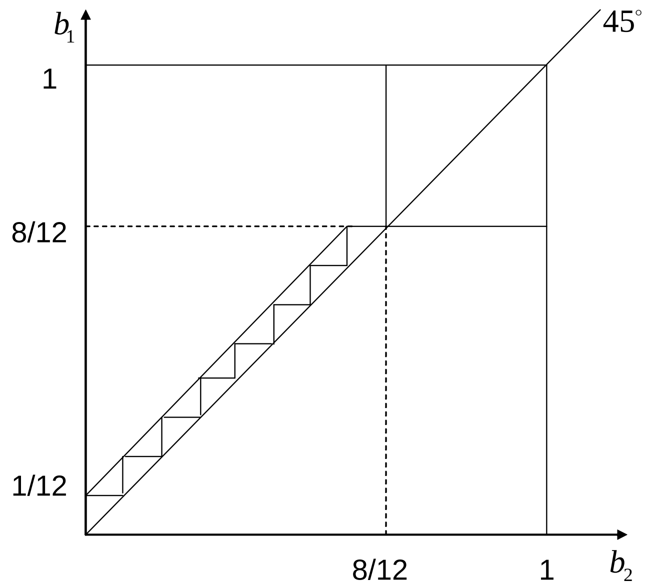

Figure 7: Eight steps of elimination of non-rationalizable strategies in the uniform price auction.

hand, if bidder 2 plays a mixed strategy which places a zero probability on the interval $[0, \frac{2}{12})$, then any strategy $b_1 \in [0, \frac{1}{12})$ leads to a zero expected payoff for bidder 1. In this case, submitting a bid of, for instance, 3/4, would guarantee a positive payoff (bidder 2 does not bid above 1). Thus, pure strategies from the interval $[0, \frac{1}{12})$ can be eliminated as not being a best response to any strategy of bidder 2, i.e. non-rationalizable. Knowing that there are no rationalizable bids below 1/12 for each player, we can perform a second step of elimination with respect to the strategies from the interval $[\frac{1}{12}, \frac{2}{12})$. Playing $\frac{2}{12}$ guarantees a strictly higher payoff when bidder 2 uses a (mixed) strategy placing a positive probability on the interval $[\frac{1}{12}, \frac{2}{12})$; and playing 3/4 gives a strictly higher payoff when bidder 2 uses a (mixed) strategy that does not place a positive probability on that interval.

With this iterative procedure we can perform eight steps of elimination by subsequently deleting the strategies from the intervals $[0, \frac{1}{12})$, $[\frac{1}{12}, \frac{2}{12})$, $[\frac{2}{12}, \frac{3}{12})$,..., $[\frac{7}{12}, \frac{8}{12})$ (see Figure 7 for an illustration).

Further, direct inspection of the payoff function shows that $b_1 = 0.69$ strictly dominates all pure strategies from the interval $[0.85, 1]$ once strategies $b_2 > 1$ are eliminated. Using Gambit we find that no further pure strategies can be eliminated by any other strategy (pure or mixed). That is, all remaining strategies in the interval $[0.66, 0.84]$ are rationalizable.

Nash equilibria

From (2) we can derive the (pure) best response correspondence,

$$b_1^*(b_2) := \arg\max_{b_1} R_1^U(b_1, b_2) = \begin{cases} 0.5 & \text{for } b_2 < \frac{3-\sqrt{2}}{7}, \\ \{\frac{16-3\sqrt{2}}{28}, 0.5\} & \text{for } b_2 = \frac{3-\sqrt{2}}{7}, \\ \frac{3b_2+1}{4} & \text{for } b_2 \in \left(\frac{3-\sqrt{2}}{7}, 3/4\right), \\ \{\frac{11}{16}, \frac{13}{16}\} & \text{for } b_2 = 3/4, \\ \frac{b_2+2}{4} & \text{for } b_2 \in (3/4, 1.1]. \end{cases}$$

which is plotted in Figure 1. The two intersections form two asymmetric pure strategy equilibria at $(b_i, b_{-i}) = (0.77, 0.69)$, $i = 1, 2$.

In order to find mixed equilibria, we used Gambit with a strategy grid Γ and grid size 0.01. There is one symmetric mixed strategy equilibrium and 1246 asymmetric ones. The support of all Nash equilibria is contained in $[0.69, 0.81]$.

Appendix B: Translation of instructions

Welcome to our experiment. Please read those instructions carefully. They are the same for every participant. Please do not talk with other participants and remain quiet during the entire experiment. Please turn off your cell phone and don't switch it on until the end of the experiment. If you have any question, raise your hand and we will come to you.

The experiment will consist of twenty rounds. An auction with two bidders will take place in each round. You are one bidder, the second one is chosen randomly for each round. So you are bidding with somebody else in each round.

All money during this experiment is real money. At the end of the experiment all the money in your account will be paid to you at the rate 100 cent = 1 €. At the beginning of the experiment you are endowed with 400 cent.

Auction rules

Each round consists of one auction. During this auction the computer can produce and sell 0, 1 or 2 units of the same good. Each unit of this good is worth 200 cent for a bidder.

The computer has to pay production cost for each unit produced. This production cost is the same for each unit produced in one round and is chosen randomly for each round. Each number between 0 and 200 (with one decimal place) has the same probability of being chosen, so each production cost of the amount {0; 0.1; 0.2; ... ; 199.9; 200.0} has the same probability of being a certain round‘s production cost. The costs of the different rounds are totally independent of each other.

During each round the auction proceeds according to the following rules:

- Each bidder makes an offer.
- All bids from 0 to 220 cent are possible, with one decimal place {0; 0.1; 0.2; ... ; 219.9; 220.0}.
- Each bidder can buy no more than one unit in each round.
- The computer can choose,
  - either to produce 0 units and not to sell anything to anybody,
  - or to produce 1 unit and to sell it for the higher bid price to the bidder offering that price,
  - or to produce 2 units and sell one to each bidder for the lower bid price offered.
- The computer always chooses the option that maximizes its profit. Its profit equals its revenues from selling the units minus its production

cost. If two possibilities yield the same profit, the computer chooses the one with more units sold.

Here are some examples:

You are bidding 190 cent, the other bidder is willing to pay only 140 cent. Production cost turn out to be 80 cent per unit in this round. The computer makes the following calculation:

- If it produces and sells one unit at a price of 190 cent, it will make a profit of: 190 cent − 80 cent = 110 cent.

- If it produces two units and sells one to each bidder at a price of 140 cent (it can just demand the lower offer according to the rules), it will yield a profit of 140 cent minus 80 cent per unit, this means a total profit of: 2 × (140 cent − 80 cent) = 120 cent.

This example shows that the computer will be better off if it produces two units and sells them at a price of 140 cent. So that is what it is going to do.

You receive 200 cent per unit you purchased during the auction at the end of the round. So for you this round's payoff is

- 200 cent minus the price you paid for the good, if you get one unit

- 0 if you receive no unit.

So in the above example you would have made a profit of

- 200 cent − 140 cent = 60 cent

- the same is true for the other bidder.

At the end of each round you are told the value of the two bids, the amount of units sold (0, 1 or 2), this round's payoffs (yours and the second bidder's), the production cost and the balance of your account. Please write down all these data on your record sheet.

After this, a new round starts. The bidders then are matched and the production cost are chosen randomly again for each round. All rounds are independent of each other.

At the end of round twenty, please answer a short questionnaire that appears on your computer screen. After answering the questionnaire, please come to the instructors and bring your record sheet. All the money in your account will be paid to you in cash.

That's all about the rules. They are the same for every participant. If you have any question, raise your hand and we will come to you. Before starting the experiment, please answer the questions on the following page to make sure that you understood all the rules.

Test questions

1. If the price were 23 cents and the computer allocated one unit to you, what would be your payoff for this period?

2. If the price were 210 cents and the computer allocated one unit to you, what would be your payoff for this period?

3. If the price were 23 cents and the computer did not allocate a unit to you, what would be your payoff for this period?

   Assume that production cost per unit were 60 cents. Your bid is 150 cents and the other bidder bids 120 cents.

4. Calculate the computer's profit if it sells 1 unit _____ and if it sells 2 units _____.

5. How many units would the computer allocate and to whom?

6. What would be your payoff and the payoff of the other bidder when the computer chooses its better alternative?

   Assume that production cost per unit were 110 cents. Your bid is still 150 cents and the other bidder bids 120 cents.

7. Calculate the computer's profit if it sells 1 unit _____ and if it sells 2 units _____.

8. How many units would the computer allocate and to whom?

9. What would be your payoff and the payoff of the other bidder when the computer chooses its better alternative?

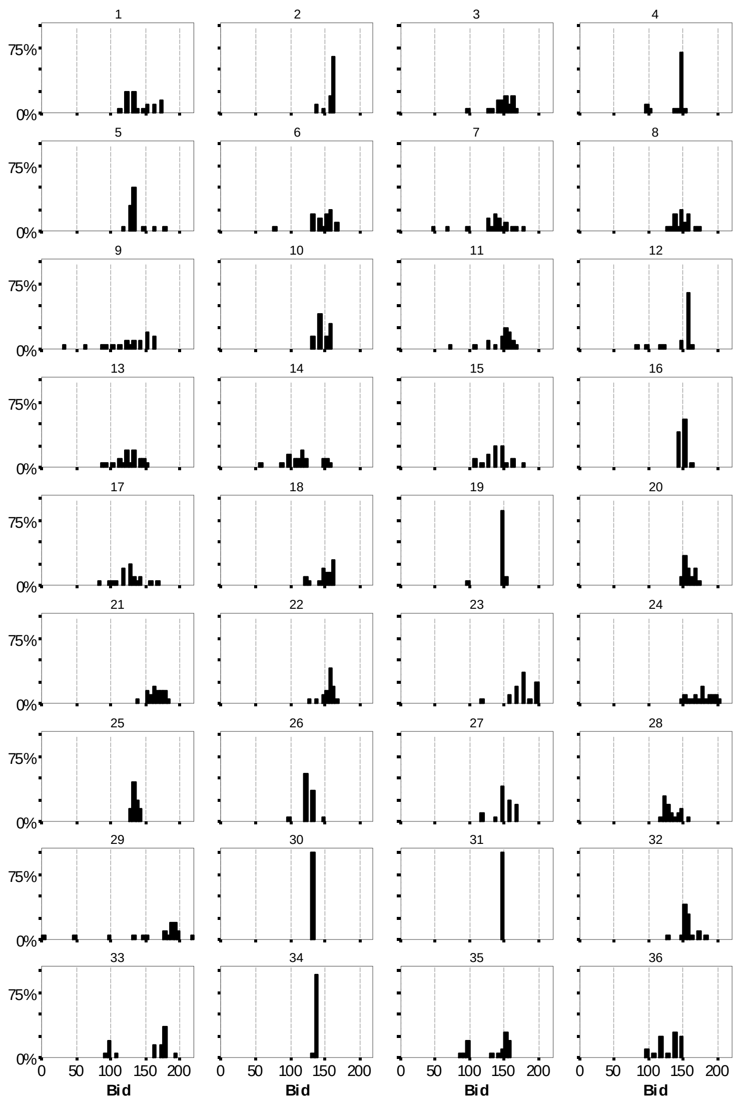

Figure 8: Distribution of bids for each individual subject in the uniform price treatment, all periods.