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You Don't Always Get What You Pay For

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Abstract

Consider a principal-agent relationship in which more effort by the agent raises the likelihood of success. Does rewarding success, i.e., paying a bonus, increase effort in this case? I find that bonuses have not only an incentive but also an income effect. Overall, bonuses paid for success may well reduce effort and hence the probability of success. I also identify conditions under which the income effect dominates the incentive effect, and single out the hazard-rate of effort as a crucial determinant of this trade-off.

Keywords: bonus, premium, incentives, income effect, moral hazard

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1 Introduction

Managers often resort to premium payments in order to boost performance above ordinary levels. Football players are promised a bonus if they win important matches, in some universities professor's budgets directly increase with each article that they publish in a reputed journal,[1] and researchers receive pay raises for successful completion of a projects. In all these cases, the outcome is not only determined by the effort of the agent but also by factors beyond his influence—that is, luck. Moreover, agents in these examples are already motivated and exert effort. Under these circumstances, threatening to *take* money in the case of failure rather than to *give* money in the case of success may induce adverse reactions by the agent and crowd out rather than increase effort (see Bewley, 1995, Fehr and Rockenbach, 2003, or Falk and Kosfeld, 2006). This is probably the reason why bonuses rather than fines are often used when agents are already motivated. The logic of bonus payments itself seems straight-forward: by paying a bonus on the condition of success, the successful outcome becomes more attractive to the agent and he puts in more effort to increase the likelihood of success. But do bonus payments necessarily increase effort and the likelihood of success?

Here, I examine this question and find that the answer is "no." There are plausible situations in which paying a bonus reduces the incentive to exert effort. For example, consider an agent who appreciates an increase of income more when he works less for it. In other words, leisure is a normal good, and richer agents derive more pleasure from exerting less effort. Then, the possibility of being lucky and receiving the bonus even if no additional effort is exerted increases the agent's income and thereby reduces his willingness to exert effort. This income effect has to be traded-off against the incentive effect of the bonus. The income effect dominates if the marginal effect of effort on the probability of success is small in relation to the probability of success when no additional effort is exerted (Proposition 1).

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[1]The university of Cologne in Germany has recently adopted such a system.

The next section introduces the model, which is then analyzed in Section 3. Section 4 summarizes the findings.

2 Model

Consider an agent who works for a principal on a project that can be successful ($S = 1$) or not ($S = 0$). Success can be verified by courts and its probability p depends on the effort e of the agent, where e is a real number. The agent likes receiving monetary transfers t. In the absence of transfers, his utility initially increases in effort until some level, $e^0 > 0$, is reached and then decreases. In other words, there is an inner solution to the agent's effort choice problem. The agent may, for example, be altruistically motivated, care about reputation or there may be other incentives that are not explicitly modeled.[2]

Let the agent's utility function $u(t, e)$ be twice continuously differentiable in transfers and effort. This specification is sufficiently flexible to account for interaction between the received transfers and effort. In particular, it may be that the agent prefers receiving a transfer when he exerts less effort. This happens, for example, if the agent is too exhausted from exerting effort to enjoy the transfer. If we regard exerting effort as the opposite of leisure, the condition is fulfilled if leisure is a normal good. There is empirical evidence that leisure is normal (see Ashenfelter and Heckman, 1974, or the survey article by Blundell and MaCurdy 1999) so it is not implausible that at least some agents enjoy transfers more when they exert less effort. In analogy to leisure being a normal good, effort is called *normal* in this case. Formally, normal effort means that the cross-partial derivative of the utility function of the agent is negative: $\frac{\partial \partial u(t,e)}{\partial t \partial e} < 0$.

The principal only cares about the expected success of the project. Standardizing the utility in case of failure to zero and in case of success to one, her utility is simply $p(e)$. She is endowed with sufficient funds to transfer money to the agent but cannot take from

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[2]Holmström and Milgrom (1991) employ a similar assumption to show that using performance measures may be detrimental in multiple-task models.

him. In other words, she can only use carrots and not sticks. This assumption reflects the idea that in many contexts, it is not possible to adversely affect the agent without risking a reduction of effort below e^0. The principal has no opportunity costs of using funds for incentives. While it may be more realistic to assume that there are opportunity costs, this assumption does not affect our results about the effect of incentives on effort.

3 Analysis

The agent is not internalizing the externality of his effort choice on the principal. It is hence straightforward that the choice of e^0 is too small from a social point of view: increased effort and an appropriate compensation of the agent could Pareto-improve the situation. Moreover, it is in the principal's self-interest to employ a mechanism that encourages the agent to exert more effort. Does such a mechanism exist?

Since effort itself cannot be contractually fixed, the principal has to rely on the project's success in order to provide incentives. More specifically, any incentive scheme for the given contractual environment is a pair of transfers (t_0, t_1), where $t_0 \geq 0$ is the transfer that the agent receives if the project is not successful and $t_1 \geq 0$ the transfer after success.[3]

In order to see whether contracts of this type can increase effort and hence yield Pareto-improvements, consider the incentive problem of the agent. Given (t_0, t_1), the agent's expected utility can be written as: $u(t_0, e) + p(e) \cdot (u(t_1, e) - u(t_0, e))$. By assumption, this utility has an inner maximizer for $t_0 = t_1 = 0$. This maximizer is given by the first-order condition:

$$\frac{\partial u(t_0, e)}{\partial e} + p'(e) \cdot (u(t_1, e) - u(t_0, e)) + p(e) \cdot \left(\frac{\partial u(t_1, e)}{\partial e} - \frac{\partial u(t_0, e)}{\partial e} \right) = 0. \quad (1)$$

Based on this condition, the implicit-function theorem can be employed to obtain the following proposition.

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[3]Transfers are non-negative because the principal cannot take from the agent.

Proposition 1. *Introducing a bonus marginally decreases effort if and only if the income effect dominates the incentive effect:*

$$p(e) \cdot (-1) \cdot \left. \frac{\partial \partial u(t,e)}{\partial e \partial t} \right|_{t=0, e=e^0} > p'(e) \cdot \left. \frac{\partial u(t,e)}{\partial t} \right|_{t=0, e=e^0}. \tag{2}$$

Proof. Applying the implicit-function theorem to (1) yields:

$$\frac{de}{dt_0} = -(\text{SOC})^{-1} \cdot \left(-p'(e) \cdot \left. \frac{\partial u(t,e)}{\partial t} \right|_{t=0, e=e^0} + (1-p(e)) \cdot \left. \frac{\partial \partial u(t,e)}{\partial e \partial t} \right|_{t=0, e=e^0} \right)$$

$$\frac{de}{dt_1} = -(\text{SOC})^{-1} \cdot \left(p'(e) \cdot \left. \frac{\partial u(t,e)}{\partial t} \right|_{t=0, e=e^0} + p(e) \cdot \left. \frac{\partial \partial u(t,e)}{\partial e \partial t} \right|_{t=0, e=e^0} \right),$$

where SOC is the second-order condition, which is negative at the maximum. If Inequality (2) holds, then effort must be normal, $\frac{\partial \partial u(t,e)}{\partial e \partial t} < 0$, the derivative with respect to t_0 is negative, and the principal sets t_0 to the lowest possible level $t_0 = 0$. Likewise, Inequality (2) implies that the last term in parentheses in the second line is negative, so that the derivative of effort with respect to t_1 is also negative. Thus paying a marginal bonus, $t_1 > 0$, reduces effort below e^0. Conversely, if Inequality (2) does not hold, the derivative of effort with respect to t_1 is positive and paying a small transfer $t_1 > 0$ increases effort. $\square$

The important insight of this proposition is that a marginal increase in the bonus paid in the case of success does not necessarily increase effort. Since success is an increasing function of effort, paying for success may thus lower the probability of success.[4]

Rewriting Inequality (2) shows that the hazard-rate, $p'(e)/p(e)$, determines whether the income or the incentive effect dominates. If the change in the probability resulting from the next unit of effort is small in relation to the current probability of success, the income effect outweighs the incentive effect and effort decreases when a bonus is introduced.

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[4]The proposition describes the effect of the introduction of a small bonus on the provision of effort. If Inequality (2) holds for arbitrary transfers and efforts, effort decreases irrespective of the bonus' size.

4 Conclusion

I have examined how the introduction of a success-dependent bonus affects agents' motivation. The main finding is that a bonus may well reduce the agent's effort and hence the probability of success. Given that leisure is a normal good, paying a bonus has an income effect that may be stronger than the incentive effect. The income effect dominates whenever the hazard rate is small, that is if the effect of the next unit of effort on the probability is small in relation to the probability of success when no additional effort is exerted.

This trade-off between income and incentive effect has, to my knowledge, not yet been studied in the agency literature, although utility functions that allow for income effects have already been considered— see for example the famous analysis of the principal-agent problem by Grossman and Hart (1983). Unlike the income effect from the labor supply decision of households, the income effect here disappears if success is a perfect measure of effort. Then, the agent has no choice but to exert effort if he wants to obtain the bonus. The trade-off between income and incentives complements the well-known trade-off between insurance and incentives (Holmström, 1979, and Shavell, 1979) and yields similar predictions: a weaker link between effort and success—i.e., more noise—renders bonus payments less effective, and empirically they should occur less often under these circumstances. Notice, however, that the trade-off studied here applies even if the agent is risk-neutral.

These findings have the following practical implication: before trying to boost effort by paying for success, it is sensible to ensure that the income effect does not dominate the incentive effect. For example, one should verify that the effort of footballers, researchers, or professors actually has a strong effect on the probability of success. If results largely depend on luck, then paying bonuses may achieve the opposite of the desired effect and decrease the probability of success.

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