University of Heidelberg

Department of Economics

Discussion Paper Series | No. 409

Economic Geography and Urban Environmental Pollution

Martin Quaas
and Andreas Lange

June 2004

Economic Geography and Urban Environmental Pollution*

Martin Quaas[†] and Andreas Lange[‡]

June 13, 2004

Abstract

This paper studies Krugman's (1991) core-periphery model and extends it to include environmental pollution. We present the first analytic proof that only an even spreading of the firms over both of the two regions or a complete agglomeration of all manufacturing firms in one region are possible in Krugman's (1991) model. It is shown that, in a model including local environmental pollution, a third and more realistic type of equilibrium may occur in which most of the manufacturing firm locate in one region, but some manufacturing remains in the other.

JEL-Classifications: **R12, J61, Q20**

Keywords: Geographical Economics, Environmental Economics, Pollution, Labor Mobility

1 Introduction

In a seminal paper, Krugman (1991) developed his "core-periphrey" model, aiming to answer the questions "How far will the tendency toward geographical concentration proceed, and where will manufacturing production actually end up?" (p. 486).

*We are grateful for helpful comments from Maximilian Mihm. Support from the Research Training Group "Environmental and Resource Economics" of the Universities Heidelberg and Mannheim, financed by the German Research Foundation DFG, is gratefully acknowledged.

[†]Corresponding author. Interdisciplinary Institute for Environmental Economics, University of Heidelberg, Germany. email: quaas@eco.uni-heidelberg.de

[‡]Interdisciplinary Institute for Environmental Economics, University of Heidelberg and ZEW Centre for European Economic Research, Mannheim, Germany. email: lange@zew.de

Though able to explain under which circumstances big industrial agglomerations arise, Krugman's model is not rich enough to answer these questions. We show this in the following by proving analytically that only two distinct types of stable long-run equilibria exist in his model: a "spreading" equilibirum in which manufacturing is spread evenly among the regions and the "core-periphery" structure in which all manufacturing production is concentrated in a single region. However in order to answer the questions posed, i.e. "how far..." and "where...", it is necessary to consider equilibria between, where agglomeration is not complete. To be able to explain these more realistic types of equilibria in the model, we introduce urban environmental problems into the analysis. This enables us to explain the existence of mid-sized cities, as are to be observed all over the world.

In particular, Krugman's model explains the emergence of big manufacturing agglomerations, as observed in early nineteenth-century America as a result of falling transportation costs at the time (p. 486). This argument should hold for industrial agglomerations in Western Europe such as Manchester in the UK or the Ruhrgebiet in Germany, where a similar rapid raise in population took place in the late nineteenth century. Recently, however, a decline in the population in these agglomerations has been observed without a notable rise in transportation costs. In contrast to Krugman's original model, we are able to explain this observation in our extended framework.[1]

There have been few attempts to include environmental issues in models similar to Krugman's core-periphery model; these include Brakman et al. (1996) and van Marrewijk (2003).[2] Brakman et al. (1996) consider congestion externalities (which may be interpreted as a sort of environmental problem) in an extension of Krugman's model with the aim of explaining the stability of "small" industrial agglomerations. In a numeric analysis, they show examples of parameter constallations, for which stable agglomerations of different size exist. van Marrewijk (2003) explicitely considers environmental pollution. He concludes that the external effects of environmental pollution reduce the range of parameters for which an agglomeration equilibrium is stable compared to a model without pollution.

In the literature, there are different detailed and elaborated introductions into

[1]Of course, we do not claim that urban environmental problems are the only cause that could explain this. Among others, structural changes will certainly play an important role. These reasons, however, are much more difficult to model in Krugman's framework.

[2]Rauscher (2003) employs a similar model structure and investigates environmental problems as well, but the focus is on different questions.

the "New Economic Geography" (Fujita et al. 1999) or "Geographical Economics" (Brakman et al. 2001), which found on Krugman's 1991 article. We therefore refrain from giving such an introduction.[3] Instead, we will shortly sum up the basics of the model in section 2

In section 3, we discuss some comparative statics of the market-equlibrium for a given distribution of the manufacturing workers over the regions. We call this the "short-run" equilibrium (cf. Krugman 1991:490). Section 4 gives the formal proof that only two types of stable (long-run) equilibria exist with mobile manufacturing workers in the Krugman-model, i.e. the spreading equilibrium and the core-periphery equilibirum. In section 5, we derive the results for the model with environmental problems. Section 6 concludes.

2 The model

The following extension of Krugman's model uses the notation of the textbook-version, as presented in Fujita et al. (1999, chapter 4 and 5).[4] The model describes a "2x2x2"-economy, with two goods, two sectors of production, and two regions. The two goods are produced in one sector each, "agriculture" and "manufacturing". The two regions 1 and 2 are identical ex ante, i.e., niether region features geographical peculiarities.

Our extension is to consider the consequences of urban environmental problems. We therefore assume that the production in the manufacturing sector causes emissions as unwanted by-products, which generate local environmental pollution in the region where manufacturing takes place.

In both sectors of production, agriculture and manufacturing, the only input is (sector-specific) labor.[5] In agriculture, a single, homogenous good is produced with constant returns to scale. Without loss of generality, one unit of labor produces one unit of output. We choose the agricultural good as numeràire. Therefore, the competitive wage rate in agriculture is unity.

Following the approach of Dixit and Stiglitz (1977), we assume a continuum of dif-

[3] Recent literature surveys are Schmutzler (1999) and Roos (2003).

[4] A similar introduction can be found in Brakman, Garretsen, and van Marrewijk (2001, chapter 3 and 4).

[5] Here, we consider environmental pollution from production. From a thermodynamical point of view, this is only possible, if we consider material resources as production inputs as well. To simplify the analysis, however, we assume that these are available as a free good.

ferentiated goods in the manufacturing sector; the number $n \in I\!R$ of firms operating is endogenous. The firms differ with respect to their location in either of the regions $r \in \{1, 2\}$,[6] but they are identical with respect to their production technology: to produce q_r units of output of one variety of manufacturing good, each firm employs the amount of labor

$$l_r = F + c \cdot q_r, \tag{1}$$

c being the constant marginal costs. Since there are fixed costs F, the production in the manufacturing sector exhibits increasing returns to scale. As a consequence, each variety of manufacturing goods is produced by only one firm.

In addition to the private costs of production, described by equation (1), social costs occur in form of environmental pollution. We assume that production in the industrial sector in region $r \in \{1, 2\}$ generates an aggregate pollution E_r, which damages the consumers, but does not affect the firms. Each firm generates emissions e_r, proportional to their output q_r. Choosing units of pollution, we set $e_r = q_r$. Total pollution E_r in region r is given by the sum of emissions of all n_r symmetric firms in r:

$$E_r = \sum_{\text{firms in region } r} e_r = n_r \cdot e_r = n_r \cdot q_r. \tag{2}$$

Pollution is local in the sense that emissions of the firms in region r generate disutility for the households in r, but not for the households in the other region.

The manufacturing sector is characterized by monopolistic competition as in Dixit and Stiglitz (1977): an endogenous variety of n_r goods is produced in either region r; different varieties of manufacturing goods are imperfect substitutes in consumption. Each firm acts as a monopolist on its output marktes, taking the actions of the other firms as given. Assuming free market entry and exit, firms will have zero profits in equilibrium.

The goods produced in one region can be consumed in the other region as well. Concerning the agricultural good, it is assumed that there are no transportation costs.[7] The transportation of manufacturing goods from one region to another is costly. When one unit of a manufacturing good is shipped between the regions, only a fraction $1/T$, $T \geq 1$, arrives at the target region. These "iceberg" transportation

[6] We denote this with a subscript r.

[7] This unrealistic assumption has been relaxed in (Fujita et al. 1999, chapter 7). In the present analysis, however, no further insights can be expected from including transportation costs for the agricultural good.

costs (Samuelson 1952) imply the following: if the price of one variety of manufacturing good produced in region $r \in \{1, 2\}$ is p_r, the price p_{rs} of this good in the other region $s \neq r$ is

$$p_{rs} = p_r T, \tag{3}$$

to exclude arbitrage. Turning to the demand side of the economy, we assume that all households have identical preferences on the composite manufacturing good M_r, the agricultural good A_r, and environmental quality, which is measured by the pollution E_r. These preferences are mapped by the following utility function:[8]

$$u_r = U(M_r, A_r, E_r) = \upsilon M_r^{\mu} A_r^{1-\mu} - D(E_r) = \upsilon M_r^{\mu} A_r^{1-\mu} - \delta E_r^{\gamma}. \tag{4}$$

Here, environmental damage $D(E_r) = \delta E_r^{\gamma}$ enters the utility function in an additive-separable form. Assuming $\delta > 0$ and $\gamma > 1$ assures that marginal damage is positive and increasing. The manufacturing composite M_r is an aggregate of all varieties of manufacturing good with a constant elasticity of substitution $\sigma > 1$:

$$M_r = \left[n_1 m_{1r}^{\frac{\sigma-1}{\sigma}} + n_2 m_{2r}^{\frac{\sigma-1}{\sigma}} \right]^{\frac{\sigma}{\sigma-1}}, \tag{5}$$

where n_r is the number of firms in the manufacturing sector and, equivalently, the number of varieties of manufacturing goods in region $r \in \{1, 2\}$. m_{1r} and m_{2r} are the quantities of each variety of manufacturing good produced in region 1 and 2, respectively, and consumed in region $r \in \{1, 2\}$ by the individual under consideration. Equation (5) expresses that consumers value the different varieties of manufacturing good equally.

Concerning the incomes of the households, it is assumed that there is no intersectoral mobility and that each individual inelastically supplies one unit of labor specific to either of the sectors. Furthermore, it is assumed that workers in the agricultural sector are tied to their home region. Normalizing total population to unity, we denote the total labor supply in the agricultural sector with $1 - \mu > 0$, and assume that $(1 - \mu)/2$ agricultural workers live in each region.

In contrast, workers in the manufacturing sector can move between the regions. Manufacturing workers in region r have an incentive to move to $s \neq r$, if the achievable level of (indirect) utility u_s in s is higher than in r. In turn, they will stay in r, if their utility u_r is higher there. In that case, manufacturing workers from s will come to r. An equilibrium is reached, if the indirect utility is the same in both

[8] To simplify notation, we assume $\upsilon = (1 - \mu)^{\mu-1} \mu^{-\mu}$.

regions, i.e. $u_1 = u_2$, or if no further migration is possible because all manufacturing workers already live in one of the regions.
In our analysis, we proceed in the following steps: (i) We analyze the (short-run) market equilibria for a given distribution of manufacturing workers in section 3. (ii) We derive conditions for the (long-run) migration equilibria with and without environmental problems in sections 4 and 5, respectively, and discuss the properties of these equilibria in detail.

3 Short-run Equilibrium

We begin the analysis by determining the short-run equilibrium, i.e. the market equilibrium for a given distribution of manufacturing workers over the two regions. Because of the additively-separable form of the utility function (4) and because there is no direct effect of the household's decisions on environmental quality, the demand functions do not depend on the environmental quality. Neither do the firm's decisions depend on their emissions, as they are not affected by the pollution and we do not consider any form of environmental policy here. Therefore, the conditions for the short-run equilibrium are the same in our model as in Krugman's original model.[9] We therefore shall be very brief in the derivation of these conditions.
Assume that $\mu \cdot \lambda$ people work in the manufacturing sector in region 1 and $\mu \cdot (1-\lambda)$ work in region 2, respectively. Let w_r be the nominal wage rate in region $r \in \{1, 2\}$. Then aggregate income Y_r is (cf. Krugman 1991, equations (15) and (16))

$$Y_1 = \mu \cdot \lambda \cdot w_1 + \frac{1-\mu}{2} \qquad \text{in region 1 and} \qquad (6)$$

$$Y_2 = \mu \cdot (1-\lambda) \cdot w_2 + \frac{1-\mu}{2} \qquad \text{in region 2.} \qquad (7)$$

The price index for the manufacturing composite is (cf. Krugman 1991, equations (17) and (18)):

$$G_1 = \left[\lambda w_1^{1-\sigma} + (1-\lambda)\left[w_2 T\right]^{1-\sigma}\right]^{\frac{1}{1-\sigma}} \qquad \text{in region 1 and} \qquad (8)$$

$$G_2 = \left[\lambda (w_1 T)^{1-\sigma} + (1-\lambda) w_2^{1-\sigma}\right]^{\frac{1}{1-\sigma}} \qquad \text{in region 2.} \qquad (9)$$

The the nominal wage rates in both regions are given by (cf. Fujita et al. 1999,

[9] By contrast, the allocation in the long-run depends on the environmental quality, as it affects the location decision of the households. This is discussed extensively in sections 4 and 5.

equations (5.11) and (5.12))

$$w_1 = \left[Y_1 G_1^{\sigma-1} + Y_2 T^{1-\sigma} G_2^{\sigma-1}\right]^{\frac{1}{\sigma}} \qquad \text{in region 1 and} \qquad (10)$$

$$w_2 = \left[Y_1 T^{1-\sigma} G_1^{\sigma-1} + Y_2 G_2^{\sigma-1}\right]^{\frac{1}{\sigma}} \qquad \text{in region 2.} \qquad (11)$$

In general, an analytic solution of the system of equations is not possible. But it is possible to derive several analytical results that provide an insight into the structure of the solution. To keep notation simple, let k denote the ratio of the nominal wage rate w_2 in region 2 to the nominal wage rate w_1 in region 1:

$$k \equiv \frac{w_2}{w_1}.$$

Furthermore, we use the abbreviations $t := T^{1-\sigma}$,[10] and

$$k_0 = \left[\frac{2t}{(1+\mu)t^2 + 1 - \mu}\right]^{\frac{1}{\sigma}}. \qquad (12)$$

We show in section A.1 of the appendix that the relevant information needed to determine the relative nominal wage rate k from the equations (6) to (11) is captured by the following single equation:

$$(\lambda - (1-\lambda)k)k_0^{-\sigma} = \lambda k^{\sigma} - (1-\lambda)k^{1-\sigma}. \qquad (13)$$

From this equation, the interpretation of k_0 becomes clear: It is the relative wage rate between regions 2 and 1, when all manufacturing workers live in 2, i.e. $k_0 = k|_{\lambda=0}$. The different values of k_0 for different sets of parameters lead to insights in different qualitative interrelations between k and λ. The results are given in the following proposition.

Proposition 1

1. *For $T > 1$, $\sigma > 1$ and $\mu < 1$, the following holds: If and only if the parameters T, σ, and μ fulfill the condition $T^{1-\sigma} = \frac{1-\mu}{1+\mu}$, the nominal wage rate is equal in both regions, for all distributions of the manufacturing workers:*

$$T^{1-\sigma} = \frac{1-\mu}{1+\mu} \quad \Leftrightarrow \quad k \equiv 1 \quad \forall \quad 0 \leq \lambda \leq 1.$$

[10] Given the elasticity of substitution σ, t can be interpreted as a measure for the transport costs. For $T = 1$, i.e. without transportation costs, we have $t = 1$. Because $\sigma > 1$, this is the maximum of all possible t. $T \to \infty$ yields $t \to 0$, i.e. small values of t correspond to high transportation costs.

2. *If the parameters* $T > 1$, $\sigma > 1$, *and* $\mu < 1$ *fulfill the inequality* $T^{1-\sigma} < \frac{1-\mu}{1+\mu}$, *it follows that for every* $0 \leq \lambda < \frac{1}{2}$ *the nominal wage rate* w_2 *in the industrial sector in region* 2 *is smaller than in region* 1, *i.e.* $k < 1$, *and for every* $\frac{1}{2} < \lambda \leq 1$ *the nominal wage rate in region* 2 *is higher than in region* 1, *i.e.* $k > 1$*:*

$$T^{1-\sigma} < \frac{1-\mu}{1+\mu} \quad \Rightarrow \quad k < (>)1 \quad \textit{for} \quad \lambda < (>)\frac{1}{2}.$$

3. *If the parameters* $T > 1$, $\sigma > 1$, *and* $\mu < 1$ *fulfill the inequality* $T^{1-\sigma} > \frac{1-\mu}{1+\mu}$, *it follows that for every* $0 \leq \lambda < \frac{1}{2}$ *the nominal wage rate* w_2 *in the industrial sector in region* 2 *is higher than in region* 1, *i.e.* $k > 1$, *and for every* $\frac{1}{2} < \lambda \leq 1$ *the nominal wage rate in region* 2 *is smaller than in region* 1, *i.e.* $k < 1$*:*

$$T^{1-\sigma} > \frac{1-\mu}{1+\mu} \quad \Rightarrow \quad k > (<)1 \quad \textit{for} \quad \lambda < (>)\frac{1}{2}.$$

4. *The price indices suffice the following inequality*

$$G_2 < (>)G_1 \quad \textit{für} \quad \lambda < (>)\frac{1}{2}.$$

Proof: The proof is given in section A.2 of the appendix. □

If the elasticity of substitution σ and the share μ of manufacturing workers in the total population are given, proposition 1 may be interpreted as a statement of how the ratio k of the nominal wages in both regions depends on the transport costs T. If they are small, i.e. $T^{1-\sigma} > \frac{1-\mu}{1+\mu}$, we can apply part 3 of proposition 1. In this case, the nominal wage is smaller in the region with less manufacturing workers. If transport costs reach a critical value $T^{1-\sigma} = \frac{1-\mu}{1+\mu}$, the nominal wage is the same in both regions, independent of the distribution of the manufacturing workers. If the transport costs are even higher, i.e. $T^{1-\sigma} < \frac{1-\mu}{1+\mu}$, the nominal wage rate is higher in the region with less manufacturing workers.

The reason for these results lies in the fact that local demand for manufacturing goods is served more from local supply, the higher the transport costs are. On the other hand, the supply of manufacturing labor is smaller in the region with less manufacturing workers. As a consequence of the localized demand for manufacturing goods from the agricultural workers, the nominal wage is higher there, when transport costs are high. If, on the other hand, transport costs are small, the effect of the localized demand of agricultural workers becomes less important. In that case, the "home market effect" dominates: The wage rate tends to be higher in the larger market, i.e., where more industrial workers live (Krugman 1991:491).

In a similar way we can interpret proposition 1 for given transport costs and a given share μ of industrial workers. Then, for a small elasticity of substitution σ, the nominal wage is higher in the region with less manufacturing workers. For a very high σ, however, the nominal wage is higher in the region with more manufacturing workers. This is because, with a rising elasticity of substitution between the manufacturing goods, the agricultural worker's demand is served by local production at a higher degree.

Finally, proposition 1 may be interpreted for given T and σ and a changing μ: For a small share μ of manufacturing workers in the total population, the nominal wage is higher in the region, where less manufacturing workers live: The localized demand of the agricultural workers dominates the manufacturing worker's demand.

Part 4 of proposition 1 says that, independent of the exact parameter constellation, the price index is smaller in the region with less manufacturing workers. The reason for this result is that the number of firms – and, therefore, the number of industrial goods that are available without transportation costs in the respective region – is higher in the region where more manufacturing workers live. This leads to a lower price index there.

These interpretations carry over to the subsequent analysis, where we do the comparative statics of the ratio of nominal wages between the regions, with respect to the share λ of manufacturing workers living in region 1. We start with equation (13), which we rearrange in the following way:

$$\lambda_S(k) := \lambda = \left[1 + \frac{k_0^{-\sigma} - k^{\sigma}}{k \left[k_0^{-\sigma} - k^{-\sigma} \right]} \right]^{-1}. \tag{14}$$

The ratio $k = w_2/w_1$ of the nominal wages lies within the range k_0 to k_0^{-1}, where $k = k_0$ is the case if and only if $\lambda = 0$, and $k = k_0^{-1}$ if and only if $\lambda = 1$ (cf. section A.3 of the appendix).

By proposition 1, we have $k_0 < 1$ for parameter sets with $t < \frac{1-\mu}{1+\mu}$ and $k_0 > 1$ for parameter sets with $t > \frac{1-\mu}{1+\mu}$. The following lemma says that equation (14) describes a one-to-one mapping of k onto λ in the ranges $k \in \left[k_0, k_0^{-1}\right]$ for $t < \frac{1-\mu}{1+\mu}$ (respectively, $k \in \left[k_0^{-1}, k_0\right]$ for $t > \frac{1-\mu}{1+\mu}$) and $\lambda \in [0, 1]$:

Lemma 1

The function $\lambda_S(k)$ – given by equation (14) – describes, how the share λ of manufacturing workers in region 1 and the ratio $k = w_2/w_1$ of the nominal wage rates between the regions 2 and 1 are related to each other in the short-run equilibrium.

For $t \neq \frac{1-\mu}{1+\mu}$, the function $\lambda_S(k)$ has the following properties:

1. *The nominal wage rates in both regions are the same, i.e. $k = 1$, if and only if one half of the manufacturing workers live in each of the regions, i.e. $\lambda_S = 1/2$.*

2. *$k = k_0$ if and only if, all manufacturing workers live in region 1, i.e. $\lambda_S = 0$; $k = k_0^{-1}$ if and only if, all manufacturing workers live in region 2, i.e. $\lambda_S = 1$.*

3. *$\lambda_S(k)$ is strictly increasing in k for $k \in \left[k_0, k_0^{-1}\right]$, if $t < \frac{1-\mu}{1+\mu}$. It is strictly decreasing in k for $k \in \left[k_0^{-1}, k_0\right]$, if $t > \frac{1-\mu}{1+\mu}$.*

Proof: The proof is given in section A.3 of the appendix. □

By part 3 of lemma 1, equation (14) may be inverted for $t \neq \frac{1-\mu}{1+\mu}$. Therefore, the ratio k of the nominal wage rates, is uniquely determined by the distribution λ of the manufacturing workers between the two regions. Since $\lambda_S(k)$ is differentiable with respect to k, we can prove the following statement as a corollary of proposition 1 and lemma 1:

Corollary 1

1. *For the range of parameters with $t < \frac{1-\mu}{1+\mu}$ ($t > \frac{1-\mu}{1+\mu}$, respectively) the following holds: The nominal wage rate in region 2 increases (decreases) relative to the nominal wage rate in region 1, if the share λ of manufacturing workers in region 1 increases.*

2. *For the range of parameters with $t < \frac{1-\mu}{1+\mu}$ ($t > \frac{1-\mu}{1+\mu}$, respectively) the following holds: The price index G_2^μ in region 2 increases (decreases) relative to the price index G_1^μ in region 1, if the share λ of manufacturing workers in region 1 increases.*

Proof: The proof is given in section A.4 of the appendix. □

Proposition 1, lemma 1 and corollary 1 describe the qualitative properties of the short-run equilibrium, i.e. the solutions of equations (6)–(11). We can use these results to derive the properties of the long-run equilibria, i.e. the market equilibria, which also is a migration equilibrium of manufacturing workers. Here, a "migration equilibrium" is called a state, in which none of the freely mobile manufacturing workers has an incentive to move to a region other, than where he lives.
In section 4 we analyze the long-run equilibria without environmental pollution, i.e. we consider the case $\delta = 0$. In section 5 we investigate, how the long-run equilibria

will change, if we include urban environmental problems in the analysis ($\delta > 0$). In particular, we discuss the impact of different parameters δ and γ, which describe the environmental damage function, on the stability of the different long-run equilibria.

4 Long-run equilibria without pollution

A long-run equilibrium is defined as a short-run equilibrium in which the distribution of the – freely mobile – manufacturing workers over the regions is also in equilibrium.[11]

We start our analysis of the long-rung equilibria by considering the case $\delta = 0$, i.e. the case without environmental problems. Then, our model simplifies to Krugman's original core-periphery model. In the absence of environmental problems, the indirect utility of a manufacturing worker living in region $r \in \{1, 2\}$ is equal to the real wage rate

$$\omega_r = w_r \, G_r^{-\mu}, \tag{15}$$

by deriving the optimum for a manufacturing worker with utility function (4) subject to his budget constraint $w_r = A_r + G_r \cdot M_r$.

Two types of long-run equilibria are well known in Krugman's model: The spreading equilibrium, in which an even share of the manufacturing workers lives in each of the regions, and the core-periphery equilibrium, in which all manufacturing workers live in either of the regions, leaving only the agricultural workers in the other one.

There are analytical results for these type of equilibria in the literature, which we will come back to briefly below. In particular, given the parameters σ and μ, the core-periphery equilibrium will be stable for sufficiently small transport costs T, i.e. for all transportation costs that are smaller than a critical value T_s (see Krugman 1991:496, Fujita et al. 1999:69ff). On the other hand, the spreading equilibrium is stable for sufficiently high transportation costs, i.e. for all T that are higher than a critical value T_b (see Fujita et al. 1999:71ff). In figure 1, these results are illustrated in a numerical example, where we have chosen $\sigma = 5$ and $\mu = 0.4$. Straight lines depict the values of λ in stable equilibria, dotted lines in unstable equilibria for different values of transport costs T. The figure shows that for transport costs $T \in [T_b, T_s]$, there exist three types of equilibria: The core-periphery equilibrium, the spreading equilibrium, and a third equilibrium in between, which is however unstable. For small transportation costs, i.e. $T < T_b$, there are two types of equilibria: The

[11] A long-run equilibrium is necessarily a short-run equilibrium, the converse is not true in general.

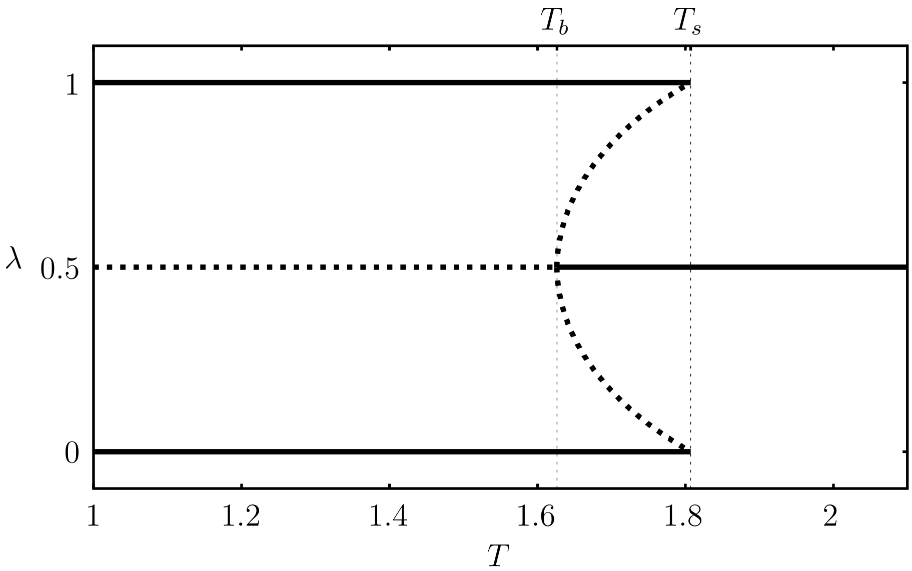

Figure 1: The 'bifurcation diagram' of the core-periphery model for parameter values $\sigma = 5$ and $\mu = 0.4$. Shown are the (long-run) equilibrium values of the share λ of manufacturing workers in region 1 for different values of transport costs T. Straight lines depict stable; dotted lines depict unstable equilibria.

core-periphery structure is stable, spreading is an unstable equilibrium. For high transport costs, i.e. $T > T_s$, spreading is the only equilibrium.

In the following, we will show analytically that figure 1 is indeed a typical example in the sense that all possible equilibria are shown.

The starting point is the definition of an interior migration equilibrium: no manufacturing worker has an incentive to move from his place of residence, i.e. $\omega_1 = \omega_2$. Inserting equation (15) and using the abbreviations $k = w_2/w_1$ and $t = T^{1-\sigma}$ again leads to the following relationship

$$\lambda_L(k) = \left[1 + k^{\sigma-1}\frac{tk^{\frac{\sigma-1}{\mu}} - 1}{t - k^{\frac{\sigma-1}{\mu}}}\right]^{-1}. \tag{16}$$

It is striking that this equation has a similar formal structure as equation (14), which determines the short-run equilibrium. This symmetry is the reason that it is possible to derive the analytic results on the stability of long-run equilibria in Krugman's model.

Lemma 2

The relation between the share λ of manufacturing workers in region 1 and the ratio $k = w_2/w_1$ of the nominal wage rates between regions 2 and 1 in an interior migra-

tion equilibrium, is given by equation (16). The function $\lambda_L(k)$ has the following properties:

1. *If $t \neq \frac{1-\mu}{1+\mu}$ and $t < 1$, the nominal wages are identical in both regions, i.e. $k = 1$, if the same share of the manufacturing workers live in both regions, i.e. $\lambda = 1/2$.*

2. *If $k = t^{-\frac{\mu}{\sigma-1}} = T^{\mu}$ ($k = t^{\frac{\mu}{\sigma-1}} = T^{-\mu}$), all manufacturing workers live in region 1 (2, i.e. $\lambda_L(T^{\mu}) = 1$ ($\lambda_L(T^{-\mu}) = 0$).*

3. *If $t < \frac{1-\mu}{1+\mu}$, $\lambda_L(k)$ is strictly increasing in k for $k \in [T^{-\mu}, T^{\mu}]$.*

Proof: The proof is given in section A.5 of the appendix. □

While equation (14) describes the relation between λ and k in the short-run equilibrium, i.e. it determines k for any given λ, equation (16) must hold for any k and λ in an interior migration equilibrium. To put it another way, equation (16) describes, how k would have to change when λ changes in order to keep the real wage in both regions at the same level.

In an interior long-run equilibrium, both conditions, equation (14) and (16) must hold simultaneously, i.e.

$$\lambda_S(k) \quad = \quad \lambda_L(k).$$

Using the abbreviation

$$\alpha = \frac{1 - k_0^{\sigma} t}{k_0^{\sigma} - t}, \tag{17}$$

this condition may be re-written as (see section A.6 of the appendix):

$$1 - k^{\sigma} \cdot k^{\frac{\sigma-1}{\mu}} - \alpha k^{\sigma} + \alpha k^{\frac{\sigma-1}{\mu}} = 0. \tag{18}$$

The solutions to this equation determine the ratios of w_2 to w_1 in the interior long-run equilibria. Given these k, equations (14) and (16), respectively, determine the corresponding distribution of the manufacturing workers. Analyzing this equation is therefore sufficient to determine the stability properties of interior long-run equilibria. This analysis yields the following results:

Proposition 2

1. *The core-periphery structure ($\lambda = 0$ or $\lambda = 1$) is a stable equilibrium, if and only if*

$$k_0 T^{\mu} = \left[\frac{2}{2 + (1+\mu)\left[T^{2(1-\sigma)} - 1\right]}\right]^{\frac{1}{\sigma}} T^{\mu - \frac{\sigma-1}{\sigma}} > 1. \tag{19}$$

If $\mu < \frac{\sigma-1}{\sigma}$ and $t < \frac{1-\mu}{1+\mu}$ and $\mu > 0$ and $\sigma > 1$ are given, there exists a $T_s > 1$, so that for all $T < T_s$ the core-periphery equilibrium is stable.

2. *If $T > 1$, the spreading equilibrium $\lambda = 1/2$ is stable, if and only if*

$$\frac{\frac{1-\mu}{1+\mu} - T^{1-\sigma}}{\frac{1-\mu}{1+\mu} + T^{1-\sigma}} > \frac{\sigma\mu}{\sigma - 1}. \tag{20}$$

   If $\mu < \frac{\sigma-1}{\sigma}$ and $t < \frac{1-\mu}{1+\mu}$ and $\mu > 0$ and $\sigma > 1$ are given, there exists a $T_b > 1$, with $T_b < T_s$ so that for all $T > T_b$ the spreading equilibrium is stable.

3. *There is at most one long-run equilibrium with $0 < \lambda < 1/2$ and at most one with $1/2 < \lambda < 1$. It exists if and only if, conditions (19) and (20) are fulfilled simultaneously, i.e. the core-periphery* and *the spreading equilibrium are stable. If it exists, it is unstable.*

Proof: The proof is given in section A.7 of the appendix. □

Results on the stability of the core-periphery structure and on the spreading equilibrium that are similar to parts 1 and 2 of proposition 2 are known in the literature (e.g. Fujita et al. 1999, chapter 5). However, the conditions given here are different from the corresponding conditions in the literature. They therefore enable new insights. In particular, it becomes evident from condition (19) why

$$\mu \quad < \quad \frac{\sigma - 1}{\sigma} \tag{21}$$

is called a "no-black-hole" condition. If it is violated, one can see at once that inequality (19) is fulfilled for every $T > 1$, i.e. the core-periphery equilibrium is always stable. Similarly, since the left hand side of condition (20) is less than unity for $T > 1$, the spreading equilibrium can never be stable if condition (21) is violated. From condition (20) one can immediately derive a second "no-black-hole" condition:

$$T^{1-\sigma} \quad < \quad \frac{1-\mu}{1+\mu}. \tag{22}$$

If this condition is violated, the left hand side of condition (20) becomes negative; the spreading equilibrium is never stable then, for any set of parameters that suffices condition (21) and the obvious requirement $\mu > 0$. The underlying reason for this condition is given in proposition 1: If condition (22) is violated, we are in the domain of part 3 of this proposition. Then – in combination with part 4 of proposition 1 –

we see that the real wage is always higher in the region where more manufacturing workers live. Condition (22) excludes this trivial situation.
Part 3 of proposition 2 shows that at most three distinct types of equilibria exist in the core-periphery model. Indeed, there is always a set of parameters, where all three types of equilibria actually occur, because $T_b < T_s$ (cf. part 2 of the proposition). However, the non-spreading interior equilibrium can not be stable in this model.
Part 3 is therefore the most remarkable statement of the proposition. It proves that Krugman's core-periphery model is indeed not rich enough to explain the existence of mid-sized cities. This result justifies the extension of the basic model which we proposed in section 2 and now investigate in the following section.

5 Long-run equilibria with pollution

To investigate the long-run equilibria in the setting with environmental problems, we first determine the indirect utility of a manufacturing worker residing in region $r \in \{1, 2\}$. This is

$$u_r = \omega_r - D(E_r) = \omega_r - \delta\, E_r^\gamma. \tag{23}$$

Here, the environmental pollution in the short-run equilibrium in region r is:

$$E_r = \mu\, \lambda_r = \begin{cases} \mu\, \lambda & \text{in region 1} \\ \mu\, (1-\lambda) & \text{in region 2} \end{cases} \tag{24}$$

In this setting the condition for an (interior) migration equilibrium, i.e. the condition for λ in the long-run equilibrium, becomes:

$$\begin{aligned} & u_1 = u_2 \\ \Leftrightarrow \quad & \omega_1 - \delta E_1^\gamma = \omega_2 - \delta E_2^\gamma. \end{aligned} \tag{25}$$

In contrast to conditions (6) to (11) which determine the short-run market equilibrium, the environmental damage actually occurs in the condition for the migration equilibrium. Thus, it has an impact on the long-run equilibrium of the model economy.
By symmetry, spreading is an equilibrium again; the core-periphery structure, i.e. $\lambda \in \{0, 1\}$, can be an equilibrium as well.

Lemma 3

For parameters $\mu > 0$, $\sigma > 1$, $T > 1$, $\delta \geq 0$ *and* $\gamma \geq 1$,

1. *the core-periphery equilibrium ($\lambda \in \{1, 0\}$) is stable, if and only if*

$$k_0 T^\mu \geq [1 - \delta\mu^\gamma]^{-1} . \tag{26}$$

2. *the spreading equilibrium $\lambda = 1/2$ is stable, if and only if*

$$\frac{2}{\sigma - 1} Z \left[\frac{1+t}{2}\right]^{\frac{\mu}{\sigma-1}} \frac{\mu(2\sigma - 1) - Z\left[(\mu^2 + 1)\sigma - 1\right]}{\sigma - \mu Z - (\sigma - 1)Z^2} - \delta\mu^\gamma \frac{\gamma}{2^{\gamma-1}} < 0, \tag{27}$$

where $Z = \frac{1-t}{1+t}$.

Proof: The proof is given in section A.8 of the appendix. □

Consequently the stability range of both the core-periphery equilibrium and the spreading equilibrium, depends on the environmental damage as given by the damage function $D(E) = \delta\, E^\gamma$. The parameter δ may be interpreted as the weight of the environmental damage relative to consumption in the utility function. In this sense, δ measures the relative valuation of environmental quality by the individuals. Concerning δ, we have the following proposition:

Proposition 3

1. *The higher the environmental damage, i.e. the higher δ is,*
   (a) *the smaller is the set of parameter values for which the core-periphery equilibrium is stable; and*
   (b) *the bigger is the set of parameter values for which the symmetric equilibrium is stable.*
2. *For a given set of parameter values $\mu > 0$, $\sigma > 1$, $\delta > 0$ and $\gamma > 1$, there exists a (small) T_u such that spreading is the only stable equilibrium for all $T \in [1, T_u]$.*

Proof: The proof is given in section A.9 of the appendix. □

This proposition reflects the fact that local environmental problems act as a "centrifugal force", i.e., they make the agglomeration less favorable. In particular, the set of parameter values, for which agglomeration is a stable equilibrium, is strictly smaller for $\delta > 0$ than for $\delta = 0$.

Part 2 of proposition 3 shows a qualitative change in the results of the model, compared to the case without environmental damage: If $\delta > 0$, there is a range of small values of T, in which the core-periphery structure is unstable and the spreading equilibrium is stable. The reason for this result is that the driving forces of agglomeration are comparatively weak for small transportation costs. On the other hand, the centrifugal forces from the pollution are high, independent of the transportation costs. Therefore, the core-periphery structure becomes stable only, if transport costs are sufficiently high and, therefore, the centripetal forces are high enough.

From the proof of proposition 3, the following can be shown:

Corollary 2

1. *For each set of parameters $\mu > 0$, $\sigma > 1$, $\gamma > 1$ and $T \geq 1$ there exists a $\delta^* > 0$ with the following property: For all $\delta > \delta^*$, the spreading equilibrium is stable.*

2. *For each set of parameters $\mu > 0$, $\sigma > 1$ and $\gamma > 1$ there is a δ^{**} with the following property: For all $\delta > \delta^{**}$, there exists no value T of transport costs, such that the core-periphery equilibrium is stable.*

Proof: The proof is given in section A.10 of the appendix. □

Corollary 2 shows that for given parameters $\mu > 0$, $\sigma > 1$, and $\gamma > 1$ and for $\delta > \hat{\delta} = \max\{\delta^*, \delta^{**}\}$, the spreading equilibrium is stable and the core-periphery equilibrium is unstable for arbitrary values of transportation costs. Thus, if the environmental damage is sufficiently high, the same effect is generated as in Krugman's model for prohibitively high transportation costs: The polluting industry is spread evenly among the regions. In contrast to the case of very high transport costs, however, there is an exchange of manufacturing goods between the regions in equilibrium.

So far, we have only considered changes in the range of stability of the core-periphery equilibrium and the spreading equilibrium.

If environmental damage is present, but not too high, there exist another type of stable long-run equilibrium, with a "big" and a "small" agglomeration:

Proposition 4

For every parameter set $\mu > 0$, $\sigma > 1$ and $T > 1$, for which in the case without environmental pollution the spreading equilibrium is unstable and the core-periphery equilibrium is stable, a $\delta^* > 0$ and a $\gamma^* > 0$ exist, such that for all δ, γ with $\delta > \delta^*$ and $\gamma > \gamma^*$ the core-periphery structure is not an equilibrium. Rather, a stable *equilibrium exists with a "big" and a "small" agglomeration, i.e.* $0 < \lambda < \frac{1}{2}$ *(or* $\frac{1}{2} < \lambda < 1$*, respectively).*

Proof: The proof is given in section A.11 of the appendix. □

Proposition 4 describes the parameter range, in which there is an equilibrium between the centripetal forces that stabilize the agglomeration dominate the centrifugal forces from the environmental damage. Therefore, a long-run equilibrium with a partial agglomeration is stable for parameters that lie within this range.
The results from this section shall be illustrated briefly with the numerical examples shown in figure 2. There, the stability range of long-run equilibria in dependency of the transport costs T are shown for different values of the weight δ of the environmental damage. The starting point of the discussion is the bifurcation diagram with

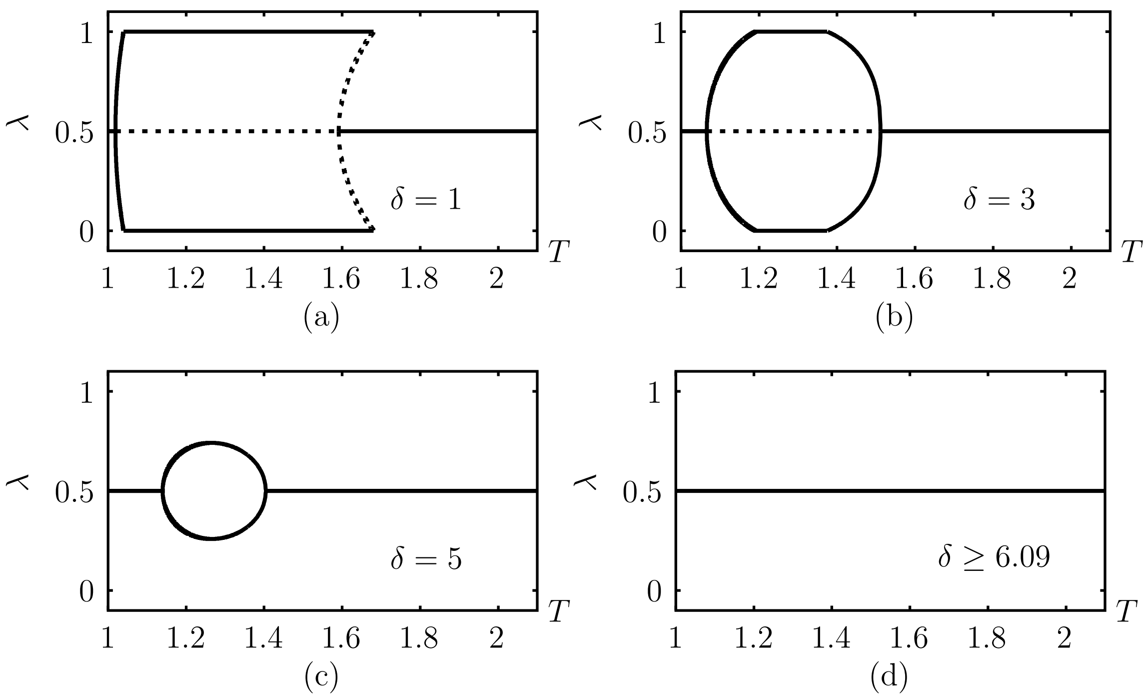

Figure 2: The bifurcation diagrams for parameters $\sigma = 5$, $\mu = 0.4$, $\gamma = 4$, and $\delta \in \{1, 3, 5\}$ as well as $\delta \geq 6.09$. Depicted are the equilibrium values of the share λ of manufacturing workers living in region 1 for varying transport costs T. Stable equilibria are depicted with straight lines, unstable equilibria with dotted lines.

vanishing environmental damage, as shown in figure 1. This may be compared to the bifurcation diagrams with different non-vanishing values of the environmental

damage δ. Comparing figure 2 (a) with figure 1 illustrates proposition 3, part 2: in the bifurcation diagram 2 (a), there is a range of small transport costs, for which the spreading equilibrium is stable, but the core-periphery structure is not. In the case of vanishing environmental damage (figure 1), such a range does not exist. Figure 2 (b) illustrates proposition 4. Here, there are two ranges of transport costs, for which the equilibria with a "big" and a "small" agglomeration, i.e. $0 < \lambda < 1/2$ and $1/2 < \lambda < 1$ are stable. In between, the core-periphery equilibrium is stable.
Figure 2 (c) gives an example for corollary 2 (a): Here, the value of δ is above the critical value $\delta^* = 3.22$, for which there is no value of transport costs, so that the core-periphery structure is stable. $\delta = 5$, finally, is above the critical value δ^{**} (which is $\delta^{**} = 6.09$ for the chosen set of parameters) where spreading is the only stable equilibrium, as depicted in figure 2 (d).
The sequence of figures 2 (a) to (d) gives an example for proposition 3: The higher the weight δ of environmental damage, the smaller is the range of transport costs, in which the core-periphery structure is a stable long-run equilibrium.

6 Conclusions and Discussion

The analysis of Krugman's (1991) core-periphery model in sections 3 and 4 has lead to some new analytical results. Proposition 1 shows that the model exhibits qualitatively different behavior for very high transport costs on the one hand, and low transport costs on the other hand. For very low transport costs, all market effects work in the same direction, the only long-run equilibrium is the core-periphery structure. For higher transport costs, the different market effects may generate different long-run equilibria, depending on the exact parameter constellation. In between, there is a critical value of transportation costs separating the sets of parameter values, where these different types of behavior occur.
Proposition 2 contains a statement in analytical form that many numerical examples have put forward before: In Krugman's core-periphery model, the only possible long-run outcomes are either the core-periphery structure or the spreading equilibrium; no further stable equilibria exist.
In order to explain the realistic structure that agglomerations of different sizes exist, it is necessary, therefore, to include additional mechanisms in the model. We have therefore introduced urban environmental problems into Krugman's model. When manufacturing production generates local pollution that harms the population living

in the respective region, a further stable equilibrium is possible, in which more than half of the manufacturing population lives in one region and the rest lives in the other one.[12]

In a further step, we have shown that the range of parameters, in which complete agglomeration is a stable equilibrium, is strictly decreasing with the weight that individuals give the environmental quality relative to consumption. We have identified parameter ranges for which the actual size of the agglomeration declines with a rising weight of environmental damage.

Taking up Krugman's (1991:286f) story that declining transport costs may explain, why large industrial agglomerations have arisen in the early nineteenth century in America and Western Europe, we are now in a position to explain why in some of them population actually declined in recent decades: While transport costs are more or less the same, the relative weight that people attach to the environmental quality of the local environment may have increased. Corresponding to our results, this would explain a decline in the large agglomeration's size.

A final remark should be made here: So far, we have argued that the decline in the size of agglomerations may be due to a change in the preferences over environmental quality. The same result can be explained in our model as a consequence of economic growth and the resulting higher importance that environmental quality gains relative to private consumption. To illustrate this, let us consider a "balanced growth path", in which all production factors grow at the same rate. In our framework, where labor is the only input, this means that the number of agricultural workers and manufacturing workers, respectively, grow at the same rate. In Krugman's original model, this has no effect – neither on the short-run nor on the long-run equilibrium – rather, it would simply imply a renormalization of the number of firms. In our model, however, environmental pollution is proportional to the number of firms. Therefore, to measure pollution correctly, any down-scaling of the number of firms requires a simultaneous up-scaling of the parameter δ, which – given γ – measures the disutility of a marginal unit of pollution. Therefore, a balanced economic growth path may be described as an increase of the parameter δ in our model: Given an appropriate set of parameters, it will result in a decrease in the agglomeration's size.

[12] Brakman et al. (1996) find a similar result in the numerical analysis of a more complicated model with congestion externalities.

A Appendix

A.1 Derivation of equation (13)

We start with the following equations, which result from equations (8), (9), (10) and (11) after inserting $t = T^{1-\sigma}$ and $k = w_2/w_1$:

$$G_1^{1-\sigma} = w_1^{1-\sigma}\left[\lambda + t\,(1-\lambda)k^{1-\sigma}\right] \tag{28}$$

$$G_2^{1-\sigma} = w_1^{1-\sigma}\left[t\,\lambda + (1-\lambda)k^{1-\sigma}\right] \tag{29}$$

$$w_1^{\sigma} = Y_1 G_1^{\sigma-1} + t\,Y_2 G_2^{\sigma-1} \tag{30}$$

$$[w_1 k]^{\sigma} = t\,Y_1 G_1^{\sigma-1} + Y_2 G_2^{\sigma-1}. \tag{31}$$

Inserting the first two equations into the last two, and inserting equations (6) and (7) yields, with some rearrangement:

$$[1 - \mu\lambda X - \mu(1-\lambda)ktY]\,\frac{2w_1}{1-\mu} \;=\; X + tY \tag{32}$$

$$[k^{\sigma} - \mu\lambda tX - \mu(1-\lambda)kY]\,\frac{2w_1}{1-\mu} \;=\; tX + Y, \tag{33}$$

with the abbreviations $X = [\lambda + (1-\lambda)tk^{1-\sigma}]^{-1}$ and $Y = [\lambda t + (1-\lambda)k^{1-\sigma}]^{-1}$. Dividing (32) by (33) leads to:

$$\begin{aligned}
& tX + Y - \mu\lambda tX^2 - \mu(1-\lambda)kt^2XY - \mu\lambda XY - \mu(1-\lambda)ktY^2 \\
& = k^{\sigma}X + k^{\sigma}Yt - \mu\lambda tX^2 - \mu(1-\lambda)kXY - \mu\lambda t^2XY - \mu(1-\lambda)ktY^2
\end{aligned}$$

$$\Leftrightarrow \quad tX + Y + \mu(1-\lambda)kXY(1-t^2) = k^{\sigma}X + k^{\sigma}tY + \mu\lambda XY(1-t^2)$$

$$\Leftrightarrow \quad \frac{t-k^{\sigma}}{Y} + \frac{1-tk^{\sigma}}{X} = \mu(1-t^2)(\lambda - k + \lambda k).$$

Re-inserting X and Y and using the abbreviation

$$k_0 = \left[\frac{2t}{(1+\mu)t^2 + 1 - \mu}\right]^{\frac{1}{\sigma}} \tag{12}$$

leads, after rearrangement, to equation

$$(\lambda - (1-\lambda)k)k_0^{-\sigma} = \lambda k^{\sigma} - (1-\lambda)k^{1-\sigma}. \tag{13}$$

A.2 Proof of Proposition 1

Ad 1: We start with equation 13.

$$
\begin{aligned}
& & (\lambda-(1-\lambda)k)k_0^{-\sigma} &= \lambda k^{\sigma}-(1-\lambda)k^{1-\sigma} \quad \text{for } k\equiv 1\\
\Leftrightarrow & & k_0^{-\sigma} &= 1\\
\Leftrightarrow & & k_0=\left[\frac{2t}{(1+\mu)t^2+1-\mu}\right]^{\frac{1}{\sigma}} &= 1\\
\Leftrightarrow & & \frac{2t}{(1+\mu)t^2+1-\mu} &= 1\\
\Leftrightarrow & & 2t &= (1+\mu)t^2+1-\mu\\
\Leftrightarrow & & t^2-2\frac{t}{1+\mu}+\frac{1}{(1+\mu)^2} &= \frac{1}{(1+\mu)^2}-\frac{1-\mu}{1+\mu}\\
\Leftrightarrow & & \left[t-\frac{1}{1+\mu}\right]^2 &= \left[\frac{\mu}{1+\mu}\right]^2\\
\Leftrightarrow & & \frac{t(1+\mu)-1}{1+\mu} = \frac{\mu}{1+\mu} \quad &\text{or} \quad \frac{t(1+\mu)-1}{1+\mu} = -\frac{\mu}{1+\mu}\\
\Leftrightarrow & & t=1 \quad &\text{or} \quad t=\frac{1-\mu}{1+\mu}.
\end{aligned}
$$

Since $\mu < 1$ and by assumption $T > 1$, i.e. $t = T^{1-\sigma} < 1$, the only solution is $t = \frac{1-\mu}{1+\mu}$, which proves part 1 of the proposition. In particular, we then have $k_0 = 1$.

Ad 2 and 3: First, keep in mind that for $t \neq \frac{1-\mu}{1+\mu}$, i.e. $k_0 \neq 1$, it follows from equation (13) that

$$\text{for every } t \neq \frac{1-\mu}{1+\mu} \text{ and } \mu > 0\text{:} \quad k = 1 \quad \Leftrightarrow \quad \lambda = \frac{1}{2}.$$

This has two implications:

- For any given set of parameters no $\lambda \neq \frac{1}{2}$ exists, for which $k = 1$. As a consequence, if for any $\lambda < \frac{1}{2}$ ($\lambda > \frac{1}{2}$) $k < 1$ ($k > 1$), this has to be true for every $\lambda < \frac{1}{2}$ ($\lambda > \frac{1}{2}$).
- If for any set of parameters with $t < \frac{1-\mu}{1+\mu}$ ($t > \frac{1-\mu}{1+\mu}$) it is true that for $\lambda < \frac{1}{2}$ ($\lambda > \frac{1}{2}$) $k < 1$ ($k > 1$) this has to be true for all other sets of parameters which fulfill the same inequality, i.e. $t < \frac{1-\mu}{1+\mu}$ ($t > \frac{1-\mu}{1+\mu}$).

As a consequence, we need to prove only, that for one particular set of parameters fulfilling $t < \frac{1-\mu}{1+\mu}$, it is true that $k < 1$ for every $\lambda < \frac{1}{2}$ and $k > 1$ for every $\lambda > \frac{1}{2}$ (this will prove part 2 of the proposition) and that for a particular set of parameters fulfilling $t > \frac{1-\mu}{1+\mu}$ it is true that $k > 1$ for every $\lambda < \frac{1}{2}$ and $k < 1$ for every $\lambda > \frac{1}{2}$ (this will prove part 3 of the proposition).

We start with part 2. Therefore, we consider the special case $t \to 0$. This yields:

$$\lim_{t\to 0} k_0 = \lim_{t\to 0}\left[\frac{2t}{(1+\mu)t^2+1-\mu}\right]^{\frac{1}{\sigma}} = 0.$$

Inserting $k_0 = 0$ in equation (13) leads to:

$$\begin{aligned} && \lambda - (1-\lambda)k = 0 \\ \Leftrightarrow && k = \frac{\lambda}{1-\lambda}. \end{aligned}$$

For every $\lambda < (>)\frac{1}{2}$, the right hand side of this equation is greater (smaller) than unity, which proves part 2 of the proposition.

To prove part 3 of the proposition, we consider at $\lambda = 0$ the case $t = 1-\epsilon$, for a small, but positive ϵ. This yields:

$$\begin{aligned} k_0 &= \left[\frac{2(1-\epsilon)}{(1+\mu)(1-\epsilon)^2+1-\mu}\right]^{\frac{1}{\sigma}} \\ &= \left[\frac{2(1-\epsilon)}{(1+\mu)(1-2\epsilon+\epsilon^2)+1-\mu}\right]^{\frac{1}{\sigma}} \\ &= \left[\frac{2(1-\epsilon)}{2-\left[2\epsilon-\epsilon^2\right](1+\mu)}\right]^{\frac{1}{\sigma}} > 1 \\ &= \left[\frac{2(1-\epsilon)}{2(1-\epsilon)+2\epsilon-\left[2\epsilon-\epsilon^2\right](1+\mu)}\right]^{\frac{1}{\sigma}} > 1. \end{aligned}$$

This expression is greater than unity for sufficiently small $\epsilon > 0$, i.e. $\epsilon < \mu$, since

$$2\epsilon - \left[2\epsilon-\epsilon^2\right](1+\mu) \leq \epsilon\left[-2\mu+\epsilon(1+\mu)\right] \leq 2\epsilon(-\mu+\epsilon) < 0.$$

This proves part 3 of the proposition.

Finally, we need to prove part 4. For this, we proceed in three steps. First, we will prove the statement for all parameter sets with $t < \frac{1-\mu}{1+\mu}$. We carry out the proof for the case $\lambda < 1/2$, the case $\lambda > 1/2$ is analogous. For $\lambda < 1/2$, it follows from part 1 of proposition 1 that $k < 1$. Furthermore,

$$\begin{aligned} && 1-\lambda > \lambda \\ \Leftrightarrow && (1-\lambda)(1-t) > \lambda(1-t) \\ \Leftrightarrow && (1-\lambda)k^{1-\sigma}(1-t) > \lambda(1-t) \\ \Leftrightarrow && \lambda t + (1-\lambda)k^{1-\sigma} > \lambda + (1-\lambda)tk^{1-\sigma} \\ \Leftrightarrow && G_2 < G_1. \end{aligned}$$

We next prove the statement for the parameter sets with $t > \frac{1-\mu}{1+\mu}$. From part 3 of proposition 1, we have for every $\lambda < 1/2$ (the case $\lambda > 1/2$ is analogous):

$$\begin{aligned}
& & k &> 1 \\
&\Leftrightarrow & k^{-\sigma} &< 1 \\
&\Leftrightarrow & \left[k_0^{-\sigma} + 1\right] k^{-\sigma} &< k_0^{-\sigma} + 1 \\
&\Leftrightarrow & k_0^{-\sigma} k^{-\sigma} - 1 &< k_0^{-\sigma} - k^{-\sigma} \\
&\Leftrightarrow & \left[k_0^{-\sigma} - k^{\sigma}\right] k^{1-\sigma} &< k \left[k_0^{-\sigma} - k^{-\sigma}\right] \\
&\Leftrightarrow & \underbrace{\frac{k_0^{-\sigma} - k^{\sigma}}{k_0^{-\sigma} - k^{\sigma} + k\left[k_0^{-\sigma} - k^{-\sigma}\right]}}_{\overset{(13)}{=} 1-\lambda} k^{1-\sigma} &> k \underbrace{\frac{k_0^{-\sigma} - k^{-\sigma}}{k_0^{-\sigma} - k^{\sigma} + k\left[k_0^{-\sigma} - k^{-\sigma}\right]}}_{\overset{(13)}{=} \lambda} \\
&\Leftrightarrow & (1-\lambda)k^{1-\sigma} &> \lambda \\
&\Leftrightarrow & (1-\lambda)k^{1-\sigma}(1-t) &> \lambda(1-t) \\
&\Leftrightarrow & \lambda t + (1-\lambda)k^{1-\sigma} &> \lambda + (1-\lambda)tk^{1-\sigma} \\
&\Leftrightarrow & G_2 &< G_1.
\end{aligned}$$

Finally, we have for the parameter sets fulfilling $t = \frac{1-\mu}{1+\mu}$ that $k \equiv 1$. Using this for every $\lambda < 1/2$ (and, analogously for $\lambda > 1/2$):

$$\begin{aligned}
& & 1-\lambda &> \lambda \\
&\Leftrightarrow & (1-\lambda)(1-t) &> \lambda(1-t) \\
&\Leftrightarrow & (1-\lambda)\underbrace{k^{1-\sigma}}_{=1}(1-t) &> \lambda(1-t) \\
&\Leftrightarrow & \lambda t + (1-\lambda)k^{1-\sigma} &> \lambda + (1-\lambda)tk^{1-\sigma} \\
&\Leftrightarrow & G_2 &< G_1.
\end{aligned}$$

□

A.3 Proof of Lemma 1

Ad 1: Part 1 follows from proposition 1, parts 2 and 3.

Ad 2: It follows from equation (13) that $\lambda = 0$ for $k = k_0$ (if $t < 1$ and $t \neq \frac{1-\mu}{1+\mu}$, i.e.

$k_0 \neq 1$):

$$\begin{aligned} & (\lambda - (1-\lambda)k_0)k_0^{-\sigma} = \lambda k_0^{\sigma} - (1-\lambda)k_0^{1-\sigma} \\ \Leftrightarrow \quad & \lambda - (1-\lambda)k_0 = \lambda k_0^{2\sigma} - (1-\lambda)k_0 \\ \Rightarrow \quad & \lambda = 0, \end{aligned}$$

since $k_0 \neq 1$ for the parameter range under consideration, i.e. $t \neq \frac{1-\mu}{1+\mu}$. In the other direction, we have $\lambda = 1$ for $k = k_0^{-1}$:

$$\begin{aligned} & (\lambda - (1-\lambda)k_0^{-1})k_0^{-\sigma} = \lambda k_0^{-\sigma} - (1-\lambda)k_0^{\sigma-1} \\ \Leftrightarrow \quad & \lambda - (1-\lambda)k_0^{-1} = \lambda - (1-\lambda)k_0^{2\sigma-1} \\ \Rightarrow \quad & \lambda = 1, \end{aligned}$$

again in the parameter range $t \neq \frac{1-\mu}{1+\mu}$ and $t < 1$, i.e. $k_0^{-1} \neq 1$.

Ad 3: To prove the strict monotonicity of $\lambda(k)$, we differentiate (14) with respect to k. This yields:

$$\frac{d}{dk}\lambda_S(k) = [\lambda_S]^2 \frac{\sigma k^{\sigma}\left[k_0^{-\sigma} - k^{-\sigma}\right] + \left[k_0^{-\sigma} - k^{\sigma}\right]\left[\left[k_0^{-\sigma} - k^{-\sigma}\right] + \sigma k^{-\sigma}\right]}{k^2\left[k_0^{-\sigma} - k^{-\sigma}\right]^2}. \tag{34}$$

The denominator of the expression on the right hand side is positive, independent of the exact value of k. In the following, we show that the numerator is positive for each and every $k \in \left[k_0, k_0^{-1}\right]$, if $t < \frac{1-\mu}{1+\mu}$ (the proof that the numerator is negative for $t > \frac{1-\mu}{1+\mu}$ is analogous). Because $\lambda_S(k)$ and $d/dk\, \lambda_S(k)$ are continuous, this will prove part 3 of the lemma.

Suppose $t < \frac{1-\mu}{1+\mu}$. In this case, we have $k_0 < 1 < k_0^{-1}$ by proposition 1. Since $k > k_0$ in this parameter range and $\sigma > 1$, the numerator of (34) is:

$$\begin{aligned} & \sigma k^{\sigma}\left[k_0^{-\sigma} - k^{-\sigma}\right] + \left[k_0^{-\sigma} - k^{\sigma}\right]\left[\left[k_0^{-\sigma} - k^{-\sigma}\right] + \sigma k^{-\sigma}\right] \\ = & \sigma \underbrace{\left[\left[\frac{k}{k_0}\right]^{\sigma} - 1\right]}_{>0} + k^{-\sigma} \underbrace{\left[k_0^{-\sigma} - k^{\sigma}\right]}_{>0} \underbrace{\left[\left[\frac{k}{k_0}\right]^{\sigma} + \sigma - 1\right]}_{>0} > 0. \end{aligned}$$

$k_0^{-\sigma} - k^{\sigma} > 0$ is true, because for $k < k_0^{-1}$:

$$k_0^{-\sigma} - k^{\sigma} > k_0^{-\sigma} - k_0^{-\sigma} = 0.$$

□

A.4 Proof of Corollary 1

Part 1 follows immediately from lemma 1: As $\lambda(k)$ is differentiable and invertible, the following holds:

$$\frac{d\lambda}{dk} = \left[\frac{dk}{d\lambda}\right]^{-1}. \tag{35}$$

Part 2 of the corollary is equivalent to the statement that $\frac{d}{d\lambda}\left[\frac{G_1}{G_2}\right]^{-\mu} > (<) 0$ for $t < \frac{1-\mu}{1+\mu}$ $(t > \frac{1-\mu}{1+\mu})$. Inserting G_1 and G_2 from equations (28) and (29) and differentiating with respect to λ – considering k as a function of λ, and using the abbreviation $k' = dk/d\lambda$ leads to:

$$\begin{aligned}
\left[\frac{G_1}{G_2}\right]^{\mu} \frac{d}{d\lambda}\left[\frac{G_1}{G_2}\right]^{-\mu} &= \left[\frac{G_1}{G_2}\right]^{\mu} \frac{d}{d\lambda}\left[\frac{\lambda + (1-\lambda)tk^{1-\sigma}}{\lambda t + (1-\lambda)k^{1-\sigma}}\right]^{\frac{\mu}{\sigma-1}} \\
&= \frac{\mu}{\sigma-1}\left[\frac{1 - tk^{1-\sigma} - (1-\lambda)(\sigma-1)tk^{-\sigma}k'}{\lambda + (1-\lambda)tk^{1-\sigma}} - \frac{t - k^{1-\sigma} - (1-\lambda)(\sigma-1)k^{-\sigma}k'}{\lambda t + (1-\lambda)k^{1-\sigma}}\right] \\
&= \frac{\mu}{\sigma-1}\left[\frac{\left[1 - tk^{1-\sigma} - (1-\lambda)(\sigma-1)tk^{-\sigma}k'\right]\left[\lambda t + (1-\lambda)k^{1-\sigma}\right]}{\left[\lambda + (1-\lambda)tk^{1-\sigma}\right]\left[\lambda t + (1-\lambda)k^{1-\sigma}\right]}\right. \\
&\qquad \left. - \frac{\left[t - k^{1-\sigma} - (1-\lambda)(\sigma-1)k^{-\sigma}k'\right]\left[\lambda + (1-\lambda)tk^{1-\sigma}\right]}{\left[\lambda + (1-\lambda)tk^{1-\sigma}\right]\left[\lambda t + (1-\lambda)k^{1-\sigma}\right]}\right] \\
&= \frac{\mu}{\sigma-1}(1-t^2)k^{1-\sigma}\frac{1 + \lambda(1-\lambda)(\sigma-1)k^{-1}k'}{\left[\lambda + (1-\lambda)tk^{1-\sigma}\right]\left[\lambda t + (1-\lambda)k^{1-\sigma}\right]}.
\end{aligned}$$

Since $\sigma > 1$ and $t < 1$, the denominator of the fraction is positive for $k' = dk/d\lambda > 0$. Thus, part 2 of the corollary follows as a consequence of part 1. □

A.5 Proof of lemma 2

To derive equation (16), we start with the condition $\omega_1 = \omega_2$ for an interior migration equilibrium. This yields:

$$
\begin{aligned}
& & \omega_1 &= \omega_2 \\
\Leftrightarrow & & \left[\frac{G_1}{G_2}\right]^{-\mu} &= k \\
\Leftrightarrow & & \frac{\lambda + (1-\lambda)tk^{1-\sigma}}{\lambda t + (1-\lambda)k^{1-\sigma}} &= k^{\frac{\sigma-1}{\mu}} \\
\Leftrightarrow & & \lambda + (1-\lambda)tk^{1-\sigma} &= k^{\frac{\sigma-1}{\mu}} \left[\lambda t + (1-\lambda)k^{1-\sigma}\right] \\
\Leftrightarrow & & \lambda \left[1 - tk^{\frac{\sigma-1}{\mu}} + k^{1-\sigma}\left[k^{\frac{\sigma-1}{\mu}} - t\right]\right] &= k^{1-\sigma}\left[k^{\frac{\sigma-1}{\mu}} - t\right] \\
\Leftrightarrow & & \lambda_L(k) := \lambda &= \left[1 + k^{\sigma-1}\frac{1 - tk^{\frac{\sigma-1}{\mu}}}{k^{\frac{\sigma-1}{\mu}} - t}\right]^{-1}.
\end{aligned}
$$

Now we prove the asserted properties of the function $\lambda_L(k)$.
Ad 1: This follows by inserting $k = 1$ in (16).
Ad 2: Inserting $k = t^{-\frac{\mu}{\sigma-1}}$ in (16) yields:

$$
\lambda_L(k) \;=\; \left[1 + t^{-\mu}\frac{tt^{-1} - 1}{t - t^{-1}}\right]^{-1} = 1.
$$

Inserting $k = t^{\frac{\mu}{\sigma-1}}$ in (16) yields:

$$
\lambda_L(k) \;=\; \left[1 + k^{\mu}\frac{t \cdot t - 1}{t - t}\right]^{-1} = 0.
$$

Ad 3: We have to show that $\frac{d\lambda_L(k)}{dk} > 0$ for all $k \in [T^{-\mu}, T^{\mu}]$. Therefore, we differentiate equation (16) with respect to k:

$$\frac{d\lambda_L}{dk} = -\lambda_L^2 k^{\sigma-2}\left[(\sigma-1)\frac{1-tk^{\frac{\sigma-1}{\mu}}}{k^{\frac{\sigma-1}{\mu}}-t} - \frac{t\frac{\sigma-1}{\mu}k^{\frac{\sigma-1}{\mu}}\left[k^{\frac{\sigma-1}{\mu}}-t\right]+\left[1-tk^{\frac{\sigma-1}{\mu}}\right]\frac{\sigma-1}{\mu}k^{\frac{\sigma-1}{\mu}}}{\left[k^{\frac{\sigma-1}{\mu}}-t\right]^2}\right]$$

$$= -\lambda_L^2 k^{\sigma-2}(\sigma-1)\left[\frac{1-tk^{\frac{\sigma-1}{\mu}}}{k^{\frac{\sigma-1}{\mu}}-t} - \frac{tk^{\frac{\sigma-1}{\mu}}\left[k^{\frac{\sigma-1}{\mu}}-t\right]+\left[1-tk^{\frac{\sigma-1}{\mu}}\right]k^{\frac{\sigma-1}{\mu}}}{\mu\left[k^{\frac{\sigma-1}{\mu}}-t\right]^2}\right]$$

$$= -\lambda_L^2 k^{\sigma-2}(\sigma-1)\left[\frac{1-tk^{\frac{\sigma-1}{\mu}}}{k^{\frac{\sigma-1}{\mu}}-t} - \frac{k^{\frac{\sigma-1}{\mu}}\left[1-t^2\right]}{\mu\left[k^{\frac{\sigma-1}{\mu}}-t\right]^2}\right]$$

$$= -\lambda_L^2 k^{\sigma-2}(\sigma-1)\ \frac{\mu\left[1-tk^{\frac{\sigma-1}{\mu}}\right]\left[k^{\frac{\sigma-1}{\mu}}-t\right]-k^{\frac{\sigma-1}{\mu}}\left[1-t^2\right]}{\mu\left[k^{\frac{\sigma-1}{\mu}}-t\right]^2}$$

$$= -\lambda_L^2 k^{\sigma-2}(\sigma-1)\ \frac{(1+\mu)t^2k^{\frac{\sigma-1}{\mu}}-(1-\mu)k^{\frac{\sigma-1}{\mu}}-\mu tk^{\frac{2(\sigma-1)}{\mu}}-\mu t}{\mu\left[k^{\frac{\sigma-1}{\mu}}-t\right]^2}$$

$$= -\lambda_L^2 k^{\sigma-2}(\sigma-1)\ \frac{\left[(1+\mu)t^2-(1-\mu)\right]k^{\frac{\sigma-1}{\mu}}-\mu t\left[1+k^{\frac{2(\sigma-1)}{\mu}}\right]}{\mu\left[k^{\frac{\sigma-1}{\mu}}-t\right]^2}.$$

This last expression is positive because the numerator of the fraction is negative since we have assumed $t^2 < \frac{1-\mu}{1+\mu}$.

□

A.6 Derivation of equation (18)

Rearranging condition $\lambda_S = \lambda_L$ yields:

$$\lambda_S(k) = \lambda_L(k)$$

$$\Leftrightarrow \quad \frac{k_0^{-\sigma}-k^{\sigma}}{k\left[k_0^{-\sigma}-k^{-\sigma}\right]} = k^{\sigma-1}\frac{tk^{\frac{\sigma-1}{\mu}}-1}{t-k^{\frac{\sigma-1}{\mu}}}$$

$$\Leftrightarrow \quad \frac{1-k_0^{\sigma}k^{\sigma}}{k^{\sigma}-k_0^{\sigma}} = \frac{tk^{\frac{\sigma-1}{\mu}}-1}{t-k^{\frac{\sigma-1}{\mu}}}$$

$$\Leftrightarrow \quad \left[1-k_0^{\sigma}k^{\sigma}\right]\left[t-k^{\frac{\sigma-1}{\mu}}\right] = \left[tk^{\frac{\sigma-1}{\mu}}-1\right]\left[k^{\sigma}-k_0^{\sigma}\right]$$

$$\Leftrightarrow \quad \left[t-k_0^{\sigma}\right]\left[1-k^{\sigma}k^{\frac{\sigma-1}{\mu}}\right] = \left[1-k_0^{\sigma}t\right]\left[k^{\frac{\sigma-1}{\mu}}-k^{\sigma}\right].$$

Using the abbreviation

$$\alpha = \frac{1 - k_0^\sigma t}{k_0^\sigma - t}, \tag{17}$$

this yields equation

$$1 - k^\sigma \cdot k^{\frac{\sigma-1}{\mu}} - \alpha k^\sigma + \alpha k^{\frac{\sigma-1}{\mu}} = 0. \tag{18}$$

A.7 Proof of proposition 2

We start the poof by showing that the statement in part 3 is correct.

Ad 3: To prove the last part of proposition 2, we start with the equation:

$$\underbrace{1 - k^\sigma \cdot k^{\frac{\sigma-1}{\mu}} - \alpha k^\sigma + \alpha k^{\frac{\sigma-1}{\mu}}}_{=:f(k)} = 0, \tag{18}$$

defining the left hand side as the function $f(k)$. For the relevant parameter range, i.e. $t < \frac{1-\mu}{1+\mu}$, the domain of f is $k \in [k_0, k_0^{-1}]$, i.e. the smallest relevant k is $k_0 < 1$ (cf. lemma 1). We have:

$$\begin{aligned} f(k_0) &= 1 - k_0^\sigma \cdot k_0^{\frac{\sigma-1}{\mu}} - \alpha k_0^\sigma + \alpha k_0^{\frac{\sigma-1}{\mu}} \\ &= 1 - k_0^{\frac{\sigma-1}{\mu}} k_0^\sigma - \frac{1 - k_0^\sigma t}{k_0^\sigma - t} k_0^\sigma + \frac{1 - k_0^\sigma t}{k_0^\sigma - t} k_0^{\frac{\sigma-1}{\mu}} \\ &= \frac{1}{k_0^\sigma - t} \left[[k_0^\sigma - t] \left[1 - k_0^{\frac{\sigma-1}{\mu}} k_0^\sigma \right] + [1 - k_0^\sigma t] \left[k_0^{\frac{\sigma-1}{\mu}} - k_0^\sigma \right] \right] \\ &= \frac{1 - k_0^{2\sigma}}{k_0^\sigma - t} \left[k_0^{\frac{\sigma-1}{\mu}} - t \right]. \end{aligned}$$

The fraction on the right hand side of this equation is positive, since for $t < \frac{1-\mu}{1+\mu}$, $k_0 < 1 \Rightarrow k_0^{2\sigma} < 1$, and, by equation (12),

$$k_0^\sigma - t = \frac{2t}{(1+\mu)t^2 + 1 - \mu} - t = \frac{t\,(1-t^2)\,(1+\mu)}{(1+\mu)t^2 + 1 - \mu} > 0,$$

since $t < 1$. The expression in brackets is positive, if and only if condition (19) is fulfilled:

$$\begin{aligned} & & k_0^{\frac{\sigma-1}{\mu}} &> t \\ &\Leftrightarrow & k_0^{\frac{\sigma-1}{\mu}} &> T^{1-\sigma} \\ &\Leftrightarrow & k_0 &> T^{-\mu} \\ &\Leftrightarrow & k_0 T^\mu &> 1. \end{aligned}$$

In the spreading equilibrium $\lambda = 1/2 \;\Leftrightarrow\; k = 1$, $f(\cdot)$ has the value

$$f(1) = 1 - 1 - \alpha 1 + \alpha 1 = 0,$$

i.e., $\lambda = 1/2$ ($\Leftrightarrow k = 1$) is an interior equilibrium, equation (18) is fulfilled.
To find the interior equilibria for arbitrary $\lambda \in (0,1)$, we have to determine the values of k, where $f(k) = 0$. To determine these values, we consider the first and second derivatives of $f(\cdot)$. In order to keep the notation clean, we consider the following transformation: $x = k^{\frac{\sigma-1}{\mu}}$. Furthermore, we define $\theta = \frac{\mu\sigma}{\sigma-1}$. Then, we get a function $g(x)$ by inserting $k = x^{\frac{\mu}{\sigma-1}}$ into the definition of $f(k)$:

$$g(x) := 1 - x^{\frac{\mu\sigma}{\sigma-1}} \cdot x - \alpha\, x^{\frac{\mu\sigma}{\sigma-1}} + \alpha\, x = 1 - x^{\theta} \cdot x - \alpha\, x^{\theta} + \alpha\, x$$

Note that $f(k) = 0$ if and only if $g(x) = 0$. Therefore, we can equivalently analyze $g(x)$.

$$\begin{aligned} g'(x) &= -(1+\theta)x^{\theta} - \alpha\theta x^{\theta-1} + \alpha \\ g''(x) &= \theta x^{\theta-2}[-(1+\theta)x + \alpha(1-\theta)]. \end{aligned}$$

In the spreading equilibrium ($\lambda = 1/2 \Leftrightarrow k = 1 \Leftrightarrow x = 1$), these derivatives are:

$$\begin{aligned} g'(1) &= -(1+\theta) - \alpha\theta + \alpha = -(1+\theta) + \alpha(1-\theta) \\ g''(1) &= \theta[-(1+\theta) + \alpha(1-\theta)]. \end{aligned}$$

Both derivatives are strictly positive, if condition (20) holds:

$$\begin{aligned} & \alpha(1-\theta) > (1+\theta) \\ \Leftrightarrow \quad & \theta = \frac{\mu\sigma}{\sigma-1} < \frac{\alpha-1}{\alpha+1}. \end{aligned}$$

We will now decide between the two cases: (i) condition (20) is fulfilled and (ii) condition (20) is not fulfilled.
Case (i) is plotted on the right hand side of figure 3. In this case, it follows that $g''(x)$ is positive for all $x < 1$ as well. As a consequence, $g(x)$ is zero for exactly one $x^* \in (x_0, 1)$ if and only if $g(x_0) > 0$, where $x_0 = k_0^{\frac{\mu}{\sigma-1}}$. This is the case, if and only if condition (19) holds (see above).[13] Then, $g'(x_0)$ is negative, but $g'(x^*)$ is negative as well, i.e. $g'(x^*) < 0$ for $g(x^*) = 0$, since the minimum value of $g(x)$ is negative.
Case (ii), i.e. the case when condition (20) is violated, is plotted on the left hand side of figure 3. Turning to this case, we prove that $g(\cdot)$ is never zero for $x \in (x_0, 1)$. When (20) does not hold, we have $-(1+\theta) + \alpha(1-\theta) \leq 0$, i.e. $g'(1) \leq 0$ and $g''(1) \leq 0$.

[13]This statement has the following implication: If $g(x_0) < 0$, $g(x)$ is zero only at $x = 1$, implicating that $g'(1) > 0$. This means, that condition (20) is fulfilled, whenever (19) is violated.

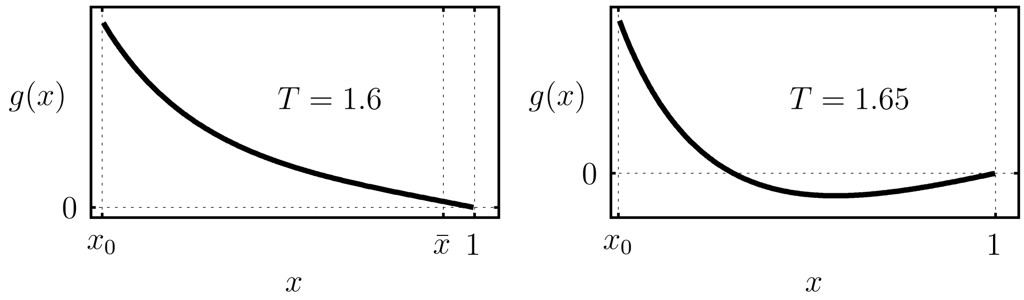

Figure 3: The function $g(x)$ in the range $x \in [x_0, 1]$. The parameters are $\sigma = 5$, $\mu = 0.4$ and $T = 1.6$ on the left hand side, or $T = 1.65$ on the right hand side.

Since $x > 0$,

$$g''(\bar{x}) = 0 \;\Leftrightarrow\; \bar{x} = \alpha \frac{1-\theta}{1+\theta}, \tag{36}$$

i.e. there exists exactly one point of inflexion $\bar{x}$ of $g(\cdot)$. At $\bar{x}$, the first derivative $g'(\cdot)$ reaches its maximum, since $g'''(\bar{x}) = -\theta \bar{x}^{\theta-2}(1+\theta) < 0$. The value of $g'(\bar{x})$ at the maximum is negative,

$$g'(\bar{x}) = -\alpha^{\theta} \frac{(1-\theta)^{\theta-1}}{(1+\theta)^{\theta-1}} + \alpha \leq 0.$$

Therefore, $g(x)$ is strictly declining in $[x_0, 1]$, i.e. $g(x) > g(1)$ for all $x \in (x_0, 1)$. This has another implication: Whenever condition (20) is violated, condition (19) is fulfilled.

Summing up: If condition (20) is fulfilled, there exists exactly one additional internal equilibrium, if and only if condition (19) is also fulfilled. However, this intermediate equilibrium is unstable, because the first derivative $g'(x)$ is negative there.

We continue the proof by showing that part 1 of the statement is valid.

<u>Ad 1:</u> We consider the case $\lambda = 0$, here. The case $\lambda = 1$ is completely symmetric. The core-periphery structure $\lambda = 0$ is a stable equilibrium, if $\omega_1/\omega_2 < 1$ for $\lambda = 0$, i.e.[14]

$$\begin{aligned} & \omega_1/\omega_2|_{\lambda=0} = \left.\frac{G_2^{\mu}}{k\, G_1^{\mu}}\right|_{\lambda=0} < 1 \\ \Leftrightarrow\quad & \frac{\left[k_0^{1-\sigma}\right]^{\frac{\mu}{1-\sigma}}}{k_0 \left[t\, k_0^{1-\sigma}\right]^{\frac{\mu}{1-\sigma}}} = \frac{1}{k_0\, t^{\frac{\mu}{1-\sigma}}} < 1 \\ \Leftrightarrow\quad & k_0\, t^{\frac{\mu}{1-\sigma}} = k_0\, T^{\mu} > 1, \end{aligned}$$

[14]It is unstable, if $\omega_1/\omega_2 > 1$ for $\lambda = 0$. We show at the end of the proof that it is unstable for $\omega_1/\omega_2 = 1$ at $\lambda = 0$, too.

where we have used equations (8), (9), $k(\lambda = 0) = k_0$, and $t = T^{1-\sigma}$. Using the definition of k_0, i.e. equation (12), this inequality reads

$$\left[\frac{2T^{1-\sigma}}{(1+\mu)T^{2(1-\sigma)}+1-\mu}\right]^{\frac{1}{\sigma}} T^{\mu} = \left[\frac{2}{2+(1+\mu)\,\left[T^{2\,(1-\sigma)}-1\right]}\right]^{\frac{1}{\sigma}} T^{\mu-\frac{\sigma-1}{\sigma}} > 1. \quad (37)$$

Our next step is to prove that exactly one $T_s > 1$ exists, which solves the equation

$$\left[\frac{2T^{1-\sigma}}{(1+\mu)T^{2(1-\sigma)}+1-\mu}\right]^{\frac{1}{\sigma}} T^{\mu} = 1.$$

We therefore consider the left hand side of this equation for $T = 1$. In this case, the left hand side is equal to 1 and the equation is fulfilled. For the case of infinitely high transport costs, i.e. $T \to \infty$, the left hand side becomes zero (given that $\mu < \frac{\sigma-1}{\sigma}$), the equation is not fulfilled. Note that the core-periphery equilibrium is then unstable. The last step is to prove that the left hand side of (37) has only one

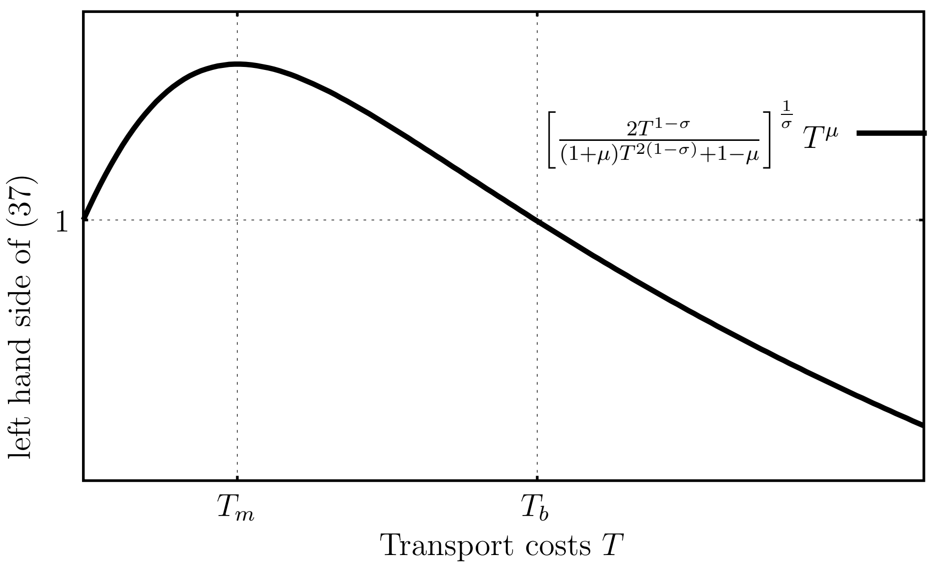

Figure 4: The left hand side of (37) for $\sigma = 5$ and $\mu = 0.4$.

interior local extremum which is a local maximum at $T = T_m > 1$. Therefore, we differentiate the left hand side of (37) with respect to T and set it equal to zero:

$$
\begin{aligned}
& 0=\frac{d}{dT}\left[\frac{2T^{1-\sigma}}{(1+\mu)T^{2(1-\sigma)}+1-\mu}\right]^{\frac{1}{\sigma}}T^{\mu} \\
\Leftrightarrow\quad & 0=\frac{d}{dT}\frac{2T^{\mu\sigma-(\sigma-1)}}{(1+\mu)T^{2(1-\sigma)}+1-\mu} \\
\Leftrightarrow\quad & 0=T^{\mu\sigma-\sigma}\frac{2(\mu\sigma-(\sigma-1))\left[(1+\mu)T^{2(1-\sigma)}+1-\mu\right]-4(1-\sigma)(1+\mu)T^{2(\sigma-1)}}{\left[(1+\mu)T^{2(1-\sigma)}+1-\mu\right]^2} \\
\Leftrightarrow\quad & 0=(\mu\sigma+(\sigma-1))(1+\mu)T^{2(1-\sigma)}+(\mu\sigma-(\sigma-1))(1-\mu) \\
\Leftrightarrow\quad & T^{2(1-\sigma)}=\frac{(\sigma-1-\mu\sigma)(1-\mu)}{(\sigma-1+\mu\sigma)(1+\mu)}\quad\Leftrightarrow\quad T_m=\left(\frac{(\sigma-1-\mu\sigma)(1-\mu)}{(\sigma-1+\mu\sigma)(1+\mu)}\right)^{\frac{1}{2(1-\sigma)}}.
\end{aligned}
$$

It is easily confirmed that $T_m > 1$, because $\sigma - 1 > \mu\sigma$ and $\mu < 1$. Furthermore, the left hand side of (37) is indeed at maximum for $T = T_m$, because at $T = 1$,

$$
\begin{aligned}
& \frac{d}{dT}\left[\frac{2T^{1-\sigma}}{(1+\mu)T^{2(1-\sigma)}+1-\mu}\right]^{\frac{1}{\sigma}}T^{\mu}\Bigg|_{T=1} \\
&=\frac{1}{\sigma}\left[\frac{2}{(1+\mu)+1-\mu}\right]^{\frac{1}{\sigma}}\frac{2(\mu\sigma-(\sigma-1))\left[(1+\mu)+1-\mu\right]-4(1-\sigma)(1+\mu)}{\left[(1+\mu)+1-\mu\right]^2} \\
&=\frac{1}{\sigma}\frac{4(\mu\sigma-(\sigma-1))-4(1-\sigma)(1+\mu)}{4}=\frac{2\mu\sigma-\mu}{\sigma}>0
\end{aligned}
$$

Since the inequality (37) holds for $T \in (1, T_m)$ and does not for $T \to \infty$, this proves that there must be exactly one $T_s > T_m > 1$ so that for every $T \in (1, T_s)$ the inequality (37) holds.

Ad 2: We now aim to derive the condition for the stability of the spreading equilibrium, equation (20). As a first step, we derive a general condition for the stability of interior equilibria.

We start with equation (16). The locus of (16) maps all the combinations of shares λ of manufacturing workers living in region 1 and ratios k of nominal wage changes, for which the real wages in both regions would be equal. This curve is plotted with a solid line in figure 5. We will see that in the neighborhood of internal equilibria, and the spreading equilibrium in particular, for all points above this curve the real wage in region 1 exceeds that in region 2 and vice versa. To see this, we repeat the

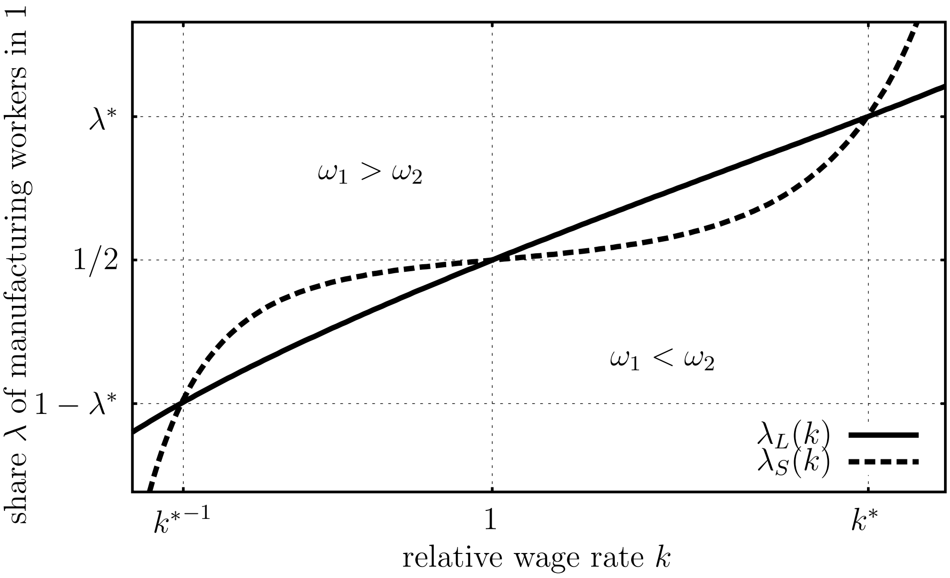

Figure 5: The share λ of manufacturing workers in 1 in the short-run equilibrium and the long-run equilibrium, respectively. The parameters are $\sigma = 5$, $\mu = 0.4$ and $T = 1.75$.

derivation of (16) (section A.5 on page 27) starting with an inequality.

$$\begin{aligned} & \omega_1 > \omega_2 \\ \Leftrightarrow \quad & \left[\frac{G_1}{G_2}\right]^{-\mu} > k \\ \Leftrightarrow \quad & \frac{\lambda + (1-\lambda)tk^{1-\sigma}}{\lambda t + (1-\lambda)k^{1-\sigma}} > k^{\frac{\sigma-1}{\mu}} \\ \Leftrightarrow \quad & \lambda + (1-\lambda)tk^{1-\sigma} > k^{\frac{\sigma-1}{\mu}}\left[\lambda t + (1-\lambda)k^{1-\sigma}\right] \\ \Leftrightarrow \quad & \lambda\left[1 - tk^{\frac{\sigma-1}{\mu}} + k^{1-\sigma}\left[k^{\frac{\sigma-1}{\mu}} - t\right]\right] > k^{1-\sigma}\left[k^{\frac{\sigma-1}{\mu}} - t\right]. \end{aligned}$$

We consider this last inequality for two cases and assume that either (i) the parameters fulfill $t < \frac{1-\mu}{1+\mu}$ ($\Rightarrow k_0 < 1$ is the minimum of all k) and $k_0^{\frac{\sigma-1}{\mu}} > t$ (i.e. the core-periphery structure is stable) or (ii) the equilibrium under consideration is the spreading equilibrium (i.e. $k = 1$) then the following holds

$$\begin{aligned} & \omega_1 > \omega_2 \\ \Leftrightarrow \quad & \lambda_L(k) > \left[1 + k^{\sigma-1}\frac{1 - tk^{\frac{\sigma-1}{\mu}}}{k^{\frac{\sigma-1}{\mu}} - t}\right]^{-1}. \end{aligned}$$

(Note that this does *not* mean that the spreading equilibrium is always stable!)

Here, we will concentrate on the spreading equilibrium.[15]
The locus of equation (14) (plotted with a dotted line in figure 5) depicts all the combinations of λ and k for which there is a short-run equilibrium. This means that following any movement of manufacturing workers, i.e. a change in λ, the ratio k of nominal wages will react such that equation (14) is fulfilled.
This yields the following conclusion: The (internal) equilibrium under consideration is stable if and only if the dotted line in figure 5 lies above (below) the solid line, if λ is a little larger (smaller) than the equilibrium value.
An interior long-run equilibrium is stable, if[16]

$$\frac{d\lambda_S(k)}{dk} < \frac{d\lambda_L(k)}{dk}.$$

This condition evaluated at the symmetric equilibrium yields:

$$\begin{aligned}
& \left.\frac{d\lambda_S(k)}{dk}\right|_{\lambda=\frac{1}{2},\,k=1} > \left.\frac{d\lambda_L(k)}{dk}\right|_{\lambda=\frac{1}{2},\,k=1} \\
\Leftrightarrow\quad & \frac{1}{4}\frac{\sigma\left[k_0^{-\sigma}-1\right]+\left[k_0^{-\sigma}-1\right]\left[k_0^{-\sigma}-1+\sigma\right]}{\left[k_0^{-\sigma}-1\right]^2} > -\frac{1}{4}(\sigma-1)\frac{(1+\mu)t^2-(1-\mu)-2\mu t}{\mu\left[1-t\right]^2} \\
\Leftrightarrow\quad & \frac{2\sigma+k_0^{-\sigma}-1}{k_0^{-\sigma}-1} > -(\sigma-1)\left[1-\frac{1+t}{\mu(1-t)}\right] \\
\Leftrightarrow\quad & \frac{2\sigma+k_0^{-\sigma}-1+(\sigma-1)\left[k_0^{-\sigma}-1\right]}{k_0^{-\sigma}-1} > (\sigma-1)\frac{1+t}{\mu(1-t)} \\
\Leftrightarrow\quad & \frac{k_0^{-\sigma}+1}{k_0^{-\sigma}-1} > \frac{\sigma-1}{\sigma\mu}\frac{1+t}{1-t} \\
\Leftrightarrow\quad & \frac{1-k_0^{\sigma}}{1+k_0^{\sigma}}\cdot\frac{1+t}{1-t} < \frac{\sigma\mu}{\sigma-1}
\end{aligned}$$

Using the definition of k_0 (equation (12)) and $t = T^{1-\sigma}$, the left hand side of this inequality becomes

$$\begin{aligned}
\frac{1-k_0^{\sigma}}{1+k_0^{\sigma}}\cdot\frac{1+t}{1-t} &= \frac{1-\frac{2t}{(1+\mu)t^2+1-\mu}}{1+\frac{2t}{(1+\mu)t^2+1-\mu}}\cdot\frac{1+t}{1-t} \\
&= \frac{(1+\mu)t^2+1-\mu-2t}{(1+\mu)t^2+1-\mu+2t}\cdot\frac{1+t}{1-t} = \frac{(1-t)^2-\mu(1-t^2)}{(1+t)^2-\mu(1-t^2)}\cdot\frac{1+t}{1-t} \\
&= \frac{1-t-\mu(1+t)}{1+t-\mu(1-t)} = \frac{1-T^{1-\sigma}-\mu(1+T^{1-\sigma})}{1+T^{1-\sigma}-\mu(1-T^{1-\sigma})} \\
&= \frac{\frac{1-\mu}{1+\mu}-T^{1-\sigma}}{\frac{1-\mu}{1+\mu}+T^{1-\sigma}}
\end{aligned}$$

[15] We will show in part 3 of this proof that interior equilibria with $\lambda \neq 1/2$ only exist, if the core-periphery structure is stable. Consequently, the above statement holds for *any* interior equilibrium.

[16] The case where $d\lambda_S(k)/dk = d\lambda_L(k)/dk$ will be considered at the end of the proof.

As the next step, we prove that there is a T_b with the property that given μ and σ with $\mu < \frac{\sigma-1}{\sigma}$ and $t < \frac{1-\mu}{1+\mu}$, the inequality (20) is true for all $T > T_s$.
First, for $T = 1$, we have

$$\left.\frac{\frac{1-\mu}{1+\mu} - T^{1-\sigma}}{\frac{1-\mu}{1+\mu} + T^{1-\sigma}}\right|_{T=1} = \frac{\frac{1-\mu}{1+\mu} - 1}{\frac{1-\mu}{1+\mu} + 1} = \frac{-2\mu}{2} = -\mu;$$

on the other hand, for $T \to \infty$,

$$\left.\frac{\frac{1-\mu}{1+\mu} - T^{1-\sigma}}{\frac{1-\mu}{1+\mu} + T^{1-\sigma}}\right|_{T\to\infty} = 1,$$

since $\sigma > 1$. Therefore, the symmetric equilibrium is never stable if $T = 1$; it is always stable if $T \to \infty$ (provided $\mu < \frac{\sigma-1}{\sigma}$ and $t < \frac{1-\mu}{1+\mu}$). In between, the left hand side of (20) is strictly increasing in T. Thus, there must be exactly one T_s, where

$$\frac{\frac{1-\mu}{1+\mu} - T_s{}^{1-\sigma}}{\frac{1-\mu}{1+\mu} + T_s{}^{1-\sigma}} = \frac{\sigma\mu}{\sigma - 1}.$$

The statement $T_b < T_s$ will be proven together with statement 3 of the proposition.
Finally, we show (i) that the core-periphery structure is unstable, if $\omega_1/\omega_2 = 1$ at $\lambda = 0$ and (ii) that the spreading equilibrium is unstable, if $d\lambda_S(k)/dk = d\lambda_L(k)/dk$ at $\lambda = 1/2$.
(i) If $\omega_1/\omega_2 = 1$ at $\lambda = 0$, we have $k_0 T^\mu = 1$, i.e. condition (19) is violated. As a consequence of part 3 of the proposition, which was proven earlier, no equilibrium exists for $\lambda \in (0, 1/2)$. Since the spreading equilibrium is stable when (19) is violated (cf. footnote 13 on page 30), the core-periphery structure can not be stable for $k_0 T^\mu = 1$.
(ii) If

$$\frac{\frac{1-\mu}{1+\mu} - T_s{}^{1-\sigma}}{\frac{1-\mu}{1+\mu} + T_s{}^{1-\sigma}} = \frac{\sigma\mu}{\sigma - 1},$$

condition (20) is violated. Then, as a consequence of part 3 of the proposition, no equilibrium exists for $\lambda \in (0, 1/2)$, and the core-periphery is a stable equilibrium. Since $\omega_1 < \omega_2$ for all $\lambda \in (0, 1/2)$ $\omega_1 < \omega_2$ holds in the neighborhood of the spreading equilibrium. Thus, it is unstable. □

A.8 Proof of lemma 3

Ad 1: In the core-periphery equilibrium $\lambda = 1$, the endogenous variables have the following values (this can be checked by inserting these values into conditions (6) to

(11)):

$$\lambda = 1, \;\; w_1 = 1, \;\; Y_1 = \frac{1+\mu}{2}, \;\; Y_2 = \frac{1-\mu}{2}, \;\; G_1 = 1, \;\; G_2 = T, \;\; E_1 = \mu, \;\; E_2 = 0.$$

It follows that $\omega_1 = 1$ and $\omega_2 = k_1 T^{-\mu} = k_0^{-1} T^{-\mu}$, since $k_0^{-1} = w_2/w_1 = w_2$ is the value of w_2 for $\lambda = 1$.
As a consequence, the condition for a stable equilibrium at $\lambda = 1$ becomes:

$$\begin{aligned} & \omega_1 - \delta E_1^\gamma > \omega_2 - \delta E_2^\gamma \\ \Leftrightarrow \quad & 1 - \delta\mu^\gamma > k_0^{-1} T^{-\mu} \\ \Leftrightarrow \quad & k_0 T^\mu > [1 - \delta\mu^\gamma]^{-1}. \end{aligned}$$

<u>Ad 2:</u> The spreading equilibrium is stable, if and only if the indirect utility in region 1 is smaller than in region 2, if λ is slightly smaller than 1/2. By symmetry, this is the case, if and only if

$$2\frac{du(\omega, E)}{d\lambda} = 2\frac{d\omega}{d\lambda} - 2\frac{dD(E)}{d\lambda} < 0. \tag{38}$$

Here, it is

$$\left.\frac{d\omega}{d\lambda}\right|_{\lambda=1/2} = \frac{2}{\sigma - 1} Z \left(\frac{1+t}{2}\right)^{\frac{\mu}{\sigma-1}} \frac{\mu(2\sigma - 1) - Z\left((\mu^2 + 1)\sigma - 1\right)}{\sigma - \mu Z - (\sigma - 1) Z^2}, \tag{39}$$

with the abbreviation $Z = \frac{1-t}{1+t}$. The derivation of this equation may be found in Fujita et al. (1999, section 5.5) and can be left out here. Furthermore, in the spreading equilibrium we have $E_1 = E_2 = \mu/2$. This yields:

$$\left.\frac{dD(E)}{d\lambda}\right|_{\lambda=1/2} = \mu D'\left(\frac{\mu}{2}\right) = \mu \frac{1}{2} \gamma \delta \left(\frac{\mu}{2}\right)^{\gamma - 1} = \delta \mu^\gamma \frac{\gamma}{2^{\gamma-1}}. \tag{40}$$

Equations (39) and (40) together with (38) yield the condition given in lemma 3. □

A.9 Proof of proposition 3

<u>Ad 1:</u> (a) This follows immediately from the condition

$$k_0 T^\mu \geq [1 - \delta\mu^\gamma]^{-1} \tag{26}$$

for the stability of the core-periphery equilibrium: The right hand side of the inequality is strictly declining in δ.

(b) This results from condition

$$\frac{2}{\sigma-1}Z\left[\frac{1+t}{2}\right]^{\frac{\mu}{\sigma-1}}\frac{\mu(2\sigma-1)-Z\left[(\mu^2+1)\sigma-1\right]}{\sigma-\mu Z-(\sigma-1)Z^2}-\delta\mu^\gamma\frac{\gamma}{2^{\gamma-1}}<0 \qquad (27)$$

for the stability of the spreading equilibrium: The left hand side of this inequality is strictly decreasing in δ. If γ is sufficiently large, i.e. $\gamma \geq 2$, it is also strictly declining in γ.

Ad 2: The first step is to show that there is a range $1 \leq T \leq \tilde{T}$, where spreading is a stable equilibrium. For $T = 1$, the first term on the left hand side of (27) is zero, i.e.

$$\left.\frac{d\omega}{d\lambda}\right|_{\lambda=1/2}=\frac{2}{\sigma-1}Z\left[\frac{1+t}{2}\right]^{\frac{\mu}{\sigma-1}}\frac{\mu(2\sigma-1)-Z\left[(\mu^2+1)\sigma-1\right]}{\sigma-\mu Z-(\sigma-1)Z^2}=0,$$

because $Z = \frac{1-t}{1+t} = 0$ follows from $t = T^{1-\sigma} = 1$ and $T = 1$. For $T = 1$, condition (27) for the stability of the spreading equilibrium is fulfilled, if and only if $\delta > 0$. Since the derivative $d\omega/d\lambda$ is continuous, there is a whole range $1 \leq T \leq \tilde{T}$, for which spreading is a stable equilibrium, if $\delta > 0$.

Furthermore, we show that there is a range $1 \leq T \leq \hat{T}$, where the core-periphery structure is unstable. For this unstable for $T = 1$. Since the left hand side of (26) is continuous in T, there is a range $1 \leq T \leq \hat{T}$ of transport costs, for which the core-periphery structure is unstable. The proof is concluded by choosing $T_u = \min\left[\tilde{T}, \hat{T}\right]$.

□

A.10 Proof of corollary 2

Ad 1: Choose

$$\delta^* = \sup_{T>1}\frac{2^\gamma\mu^{-\gamma}Z}{\gamma(\sigma-1)}\left[\frac{1+t}{2}\right]^{\frac{\mu}{\sigma-1}}\frac{\mu(2\sigma-1)-Z\left[(\mu^2+1)\sigma-1\right]}{\sigma-\mu Z-(\sigma-1)Z^2}. \qquad (41)$$

This supremum exists, since

- For $T \to 1$ the maximand equals zero,
- for $T \to \infty$ the maximand is $\frac{2^\gamma\mu^{-\gamma}}{\gamma(\sigma-1)}\frac{\mu(2\sigma-1)-\left[(\mu^2+1)\sigma-1\right]}{\sigma-\mu+\sigma-1} < \infty$,
- the denominator of the last fraction is bounded from below by

  $$\sigma-\mu Z-(\sigma-1)Z^2 \geq \sigma-\mu+\sigma-1 = 2\sigma-1-\mu > 0$$

  and the maximand is continuous in T.

Ad 2: Choose

$$\delta^{**} = \sup_{T>1} \frac{k_0 T^\mu - 1}{\mu^\gamma k_0 T^\mu}, \tag{42}$$

where k_0 is given by equation (12) (note that k_0 depends on T). This supremum exists, since

- For $T \to 1$ the maximand equals zero,
- for $T \to \infty$ the maximand tends towards $-\infty$,
- and the maximand is continuous in T.

□

A.11 Proof of proposition 4

We start with a parameter set μ, σ, and T, for which the spreading equilibrium without environmental damage is unstable, i.e.

$$\frac{2}{\sigma-1} Z \left[\frac{1+t}{2}\right]^{\frac{\mu}{\sigma-1}} \frac{\mu(2\sigma-1) - Z\left[(\mu^2+1)\sigma - 1\right]}{\sigma - \mu Z - (\sigma-1)Z^2} > 0.$$

In the presence of an environmental damage, the spreading equilibrium is unstable, if the following is true:

$$\delta\mu^\gamma \frac{2\gamma}{2^\gamma} < \frac{2}{\sigma-1} Z \left[\frac{1+t}{2}\right]^{\frac{\mu}{\sigma-1}} \frac{\mu(2\sigma-1) - Z\left[(\mu^2+1)\sigma - 1\right]}{\sigma - \mu Z - (\sigma-1)Z^2}.$$

Furthermore, for

$$\delta\mu^\gamma \geq \frac{k_0 T^\mu}{k_0 T^\mu - 1}$$

the core-periphery structure is unstable. Now choose $\delta(\gamma) = \Delta\mu^{-\gamma}$ with $\Delta > \frac{k_0 T^\mu}{k_0 T^\mu - 1}$. Inserting this in the above condition for the (un-)stability of the spreading equilibrium, we find he condition

$$\delta(\gamma)\mu^\gamma \frac{2\gamma}{2^\gamma} = \Delta\frac{2\gamma}{2^\gamma} < \frac{2}{\sigma-1} Z \left[\frac{1+t}{2}\right]^{\frac{\mu}{\sigma-1}} \frac{\mu(2\sigma-1) - Z\left[(\mu^2+1)\sigma - 1\right]}{\sigma - \mu Z - (\sigma-1)Z^2}.$$

For $\mu > 0$, there is always a $\gamma^* > 2$, that fulfills this condition, since we have assumed that μ, σ, and T are such that the right hand side of this condition is positive. Given this γ^*, we choose $\delta^* = \delta(\gamma^*) = \Delta\mu^{\gamma^*}$. □

References

Brakman, S., H. Garretsen, R. Gigengack, C. van Marrewijk, and R. Wagenvoort (1996). Negative Feedbacks in the Economy and Industrial Location. *Journal of Regional Science*, **36**(4):631–651.

Brakman, S., H. Garretsen, and C. van Marrewijk (2001). *An Introduction to Geographical Economics*. Cambridge University Press, Cambridge.

Dixit, A. K. and J. E. Stiglitz (1977). Monopolistic Competition and Optimum Product Diversity. *American Economic Review*, **67**(3):297–308.

Fujita, M., P. Krugman, and A. J. Venables (1999). *The Spatial Economy – Cities, Regions, and International Trade*. MIT Press, Massachusetts.

Krugman, P. (1991). Increasing Returns and Economic Geography. *Journal of Political Economy*, **99**:483–499.

Rauscher, M. (2003). Hot Spots, High Smokestacks and the Geography of Pollution. Paper presented at the Workshop *Population and Environment*, Max Planck Institute for Demographic Research, Rostock.

Roos, M. (2003). Internationale Integration und die Neue Ökonomische Geographie. *Perspektiven der Wirtschaftspolitik*, **4**(1):107–120.

Samuelson, P. A. (1952). The Transfer Problem and Transport Costs: The Terms of Trade when Impediments are Absent. *Economic Journal*, **62**:278–304.

Schmutzler, A. (1999). The New Economic Geography. *Journal of Economic Surveys*, **13**(4):355–379.

van Marrewijk, C. (2003). Locational Competition and the Environment. Paper presented at the Workshop *Spatial Environmental Economics,* Interdisciplinary Institute for Environmental Economics, University of Heidelberg.