Intertemporal efficiency and equity under hyperbolic preferences. Ex ante versus ex post procrastination

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Abstract: In this paper I extend the well known result that a hyperbolically discounting agent postpones costs into the future. If society has hyperbolic intertemporal preferences, it may be optimal from an ex ante point of view to postpone structural change from a polluting to a non polluting production sector into the future (*ex ante procrastination*). The consequences of ex ante procrastination are discussed for three different behavioral patterns. I show that, depending on the assumed behavioral regime, ex ante procrastination may lead to *ex post procrastination*, i.e. de facto no investment in the non polluting sector is undertaken over the whole time horizon, although investment was optimal from an ex ante point of view. Furthermore, the ex post implemented investment plan may be inefficient if it is not dictatorial. Hence, in the case of hyperbolic preferences there is a potential trade-off between intertemporal efficiency and equity.

Keywords: dynamic optimization, emission abatement, hyperbolic preferences, intergenerational equity, intertemporal decision theory, procrastination

JEL-Classification: D91, Q25, C61

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1 Introduction

If projects undertaken today influence the future, the costs and benefits of these projects have to be aggregated and evaluated over time. The standard procedure, first introduced by Ramsey (1928) and put on an axiomatic basis by Debreu (1954) and Koopmans (1960), is to identify the welfare costs and benefits occurring at different times, discount them at a constant rate and sum them up. Hence, the weight at which future costs and benefits influence today's decisions exponentially declines over time.

This exponential standard discounting model has increasingly been challenged by empirical evidence (for an overview see Gintis 2000 and Frederick et al. 2002), which suggests that decision makers discount rather hyperbolically than exponentially, i.e. the discount rate is not a constant but declining over time. The most prominent feature of non exponential discounting is the problem of time-inconsistency, first analyzed by Strotz (1956): an intertemporal optimal consumption plan derived at time $t = 0$ is not optimal anymore if reevaluated at a later time $t > 0$. If the decision maker anticipates her own dynamic inconsistent behavior, it is rational for her to commit herself today mandatory to future actions or, if no commitment can be enforced, to play the subgame perfect Nash equilibrium of the non-cooperative sequential game with her later selves (Phelps and Pollak 1968). In the past, hyperbolic preferences have often been interpreted as irrational, due to the time-inconsistency problem.

Hence, a recent strand of economic literature on hyperbolic preferences tries to "rationalize" declining discount rates. Ahlbrecht and Weber (1995) formulate an axiomatic approach to hyperbolic discounting. Weitzman (1998) and Azfar (1999) show that risk over the future states of the world or the individual mortality risk lead to declining discount rates, even if the decision makers have constant intrinsic rates of time preference. Weitzman (2001) analyzes the aggregation of different constant individual discount rates to a declining social rate.

This paper contributes to a second strand of the literature, which takes the hyperbolic preferences of the decision maker as given and analyzes the resulting consequences. I analyze optimal structural change from a polluting to a non polluting production sector, given that the decision maker discounts hyperbolically. A ubiquitous feature in environmental economics is that the welfare costs and benefits often spread over decades or even centuries. As the optimal intertemporal decision depends the more on the discount function applied, the longer the time horizon over which the costs and benefits of the decisions spread, discounting is often the crucial issue in environmental economics (e.g. Lind 1982, Portney and Weyant 1996 and IPCC 2001a). As an example, think of the problem of climate change due to the anthropogenic greenhouse effect. The costs of CO_2 abatement, e.g. by investments in non fossil fuel based energy technologies, occur today, while the benefits spread over the succeeding centuries. Another characteristic of such long planning horizons is that not only one but a series of generations is involved. As a consequence, I model a series of non overlapping generations (each represented by a unique decision maker) where the welfare of the present generation is also influenced by the welfare of all future generations. In fact, I assume that today's welfare is the discounted sum of the welfare obtained by the present and all future generations where

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the discount rates are declining over time. This setting was first introduced by Phelps and Pollak (1968) which they called *imperfect altruism*. But in contrast to their and more recent contributions I do not restrict the analysis to the special functional form of the *quasi hyperbolic discount function* (e.g. Laibson 1997, Laibson 1998 and Harris and Laibson 2001). Recently, Chichilnisky (1996) and Li and Lofgren (2000) showed (in different model settings) that declining discount rates are consistent with a rule where current generations must also take into account the welfare of future generations.

I improve the well known result from the hyperbolic discounting literature that it can be optimal to postpone the investment into the future (e.g. Ackerlof 1991, O'Donoghue and Rabin 1999, Brocas and Carrillo 2001) by identifying two different types of delay. While *ex ante procrastination* is a fundamental feature of hyperbolic preferences, which amounts to the declining discount rates of the decision maker, *ex post procrastination* occurs because of the time-inconsistency problem and can only be observed if no mandatory commitment can be enforced. Furthermore, if no mandatory commitment can be enforced the ex post implemented plan may be Pareto-inefficient (depending on the set of exogenous given parameters and the functional form of the discount and the utility function). In this case the ex post implemented plan is either efficient but dictatorial, or non-dictatorial but inefficient. Hence, there is a potential trade-off between intertemporal efficiency and equity, which cannot occur in the case of exponential discounting.

The paper is organized as follows. In section 2 the intertemporal preferences of the decision maker and the production possibilities are introduced. The ex ante optimal plan is analyzed in section 3, while section 4 is devoted to the ex post implemented plan. Welfare and equity concerns are discussed in section 5. In section 6 numerical examples illustrate the results. Section 7 concludes.

2 The model

2.1 Intertemporal preferences and hyperbolic discounting

Assume a decision maker in each period t, *agent t*, who makes the consumption and investment decision in period t. Note that the various agents t can be identified with *one* physical person at different times t in short term problems or with *different* physical persons at different times t for the analysis of intergenerational problems. In the following I will concentrate on the second line of interpretation which is in line with the long time horizons involved in environmental economic problems. Assume further that agent t's intertemporal welfare W is her own welfare plus the discounted sum of the future welfare of her successors. Furthermore, instantaneous welfare depends on consumption c and on damage to environmental quality, which hinges upon the amount of emissions e produced:

$$W(t) = \sum_{n=t}^{\tau} D(n - t + 1) \left[U(c(t)) - S(e(t)) \right] , \tag{1}$$

where $D > 0$ denotes a discount function and τ the time horizon. U represents the instantaneous welfare gains due to consumption and S the instantaneous welfare loss as a consequence of the emissions produced. Furthermore, I suppose U and S to be twice

continuously differentiable and to exhibit standard properties (partial derivatives are indicated by subscripts):

$$U_c > 0 , \qquad U_{cc} < 0 ,$$
$$S_e > 0 , \qquad S_{ee} \geq 0 , \qquad S(0) = 0 . \tag{2}$$

Note that (1) implies that all agents apply the same discount function D. Suppose that all agents prefer own consumption to consumption of her successors and environmental damage suffered by future generations to environmental damage today. As a consequence, the discount function D is a strictly decreasing function over time. Without loss of generality, I scale the discount function to yield $D(t=1) = 1$.

If the agents have exponential intertemporal preferences, the discount function reads:

$$D^{exp}(t) = \frac{1}{(1+\rho)^{t-1}} , \tag{3}$$

where ρ denotes the *constant rate of time preference*. Hence, the following equation holds for all t:

$$\rho = \frac{D^{exp}(t)}{D^{exp}(t+1)} - 1 , \qquad \forall \, t = 1, \ldots, \tau-1 . \tag{4}$$

In the case of non-stationary intertemporal preferences the rate of time preference is not longer a constant. Nevertheless, I define the *instantaneous rate of time preference* $\rho(t)$ analogously to equation (4):

$$\rho(t) = \frac{D(t)}{D(t+1)} - 1 > 0 , \qquad \forall \, t = 1, \ldots, \tau-1 . \tag{5}$$

In the following I assume hyperbolic preferences, i.e. the sequence of $\rho(t)$ is weakly decreasing over time.

$$\rho(t) \geq \rho(t+1) , \; \forall t = 1, \ldots, \tau-1 \quad \wedge \quad \exists t = 1, \ldots, \tau-1 , \; \rho(t) > \rho(t+1) . \tag{6}$$

As shown by Strotz (1956), for all non-stationary intertemporal preferences, including hyperbolic preferences, the potential problem of time-inconsistency occurs. This means that an ex ante optimal intertemporal consumption and investment plan, derived by maximizing intertemporal welfare in period $t = 0$, will be suboptimal if reevaluated at a later date $t > 0$. If the agents are fully aware of the problem of time-inconsistency, it can be overcome by agent 1's mandatory commitment to the ex ante optimal intertemporal consumption and investment plan. In this case all successors have to stick to the ex ante optimal plan, even if they would like to alter it according to future reevaluations. If agent 1 does not have the possibility to mandatory commit her successors to future actions but anticipates their future depart from the ex ante optimal intertemporal plan, the agents end up in a non-cooperative sequential game (Phelps and Pollak 1968). Hence, the best thing to do for all agents t is playing the subgame perfect Nash equilibrium of this non-cooperative game. If the agents do not recognize that their preferences are non-stationary, in general they will alter their previously derived intertemporal plan every time they reevaluate it.

As a consequence, if we want to determine the consumption and investment plan carried out, it is not sufficient to assume that all agents maximize their intertemporal welfare. In addition, we have to specify to which extent they are aware of the potential problem of time-inconsistency, to which degree they can commit themselves mandatory to future actions and how often they are going to reevaluate the alleged optimal ex ante plan. Assuming that every agent reevaluates the former intertemporal plan of her
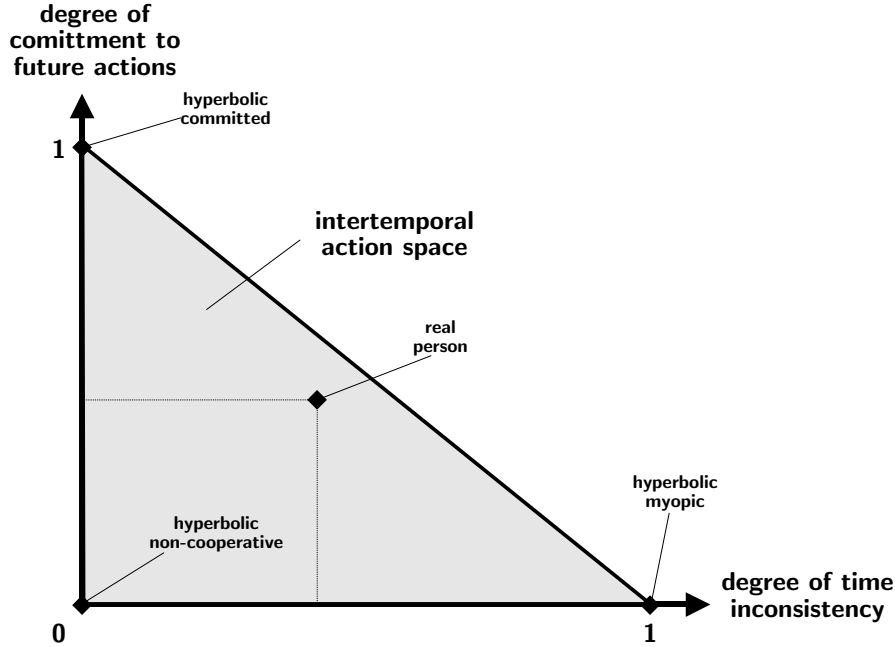

Figure 1: Sketch of the intertemporal action space for hyperbolic preferences. The corners are the three behavior patterns *hyperbolic committed*, *hyperbolic myopic* and *hyperbolic non-cooperative*. In general, the behavior of real persons with hyperbolic preferences will be described by a point inside the manifold.

predecessor and eventually modifies it, we have to further specify to what extend the agents can commit their successors to future actions and to what degree they are aware of the non-stationarity of their own preferences. As these two characteristics are at least partly independent of each other, they span a two dimensional manifold of possible intertemporal actions. Simplified, we can sketch this manifold as a triangle as done in figure 1. In the following I focus on three special behavior patterns.

Definition 1 (hyperbolic committed)
The agents are fully aware of the potential problem of time-inconsistency and agent 1 commits her successors mandatory to the intertemporal optimal ex ante consumption and investment plan. In the following I call this behavior hyperbolic committed.

Definition 2 (hyperbolic myopic)
The agents are totally unaware of the non-stationarity of their preferences. They reevaluate the ex ante optimal plan in every period and will in general modify it. This behavior

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is irrational in so far as the agents do not learn about their own and the other agents time inconsistent behavior. In the following I refer to this behavior as hyperbolic myopic.

Definition 3 (hyperbolic non-cooperative)
All agents are fully aware of the potential problem of time-inconsistency but do not have the possibility to commit their successors mandatory to the intertemporal optimal ex ante consumption and investment plan. Hence, to derive a time consistent plan they play the subgame perfect Nash equilibrium of the non-cooperative sequential game against all other agents. In the following this behavior is termed hyperbolic non-cooperative.

In fact, these three behavior patterns correspond with the corners of the intertemporal action space and have to be assessed as the three possible extreme cases. Real persons, who have hyperbolic preferences, are more likely to exhibit a behavior pattern, which may be described by an interior point of the intertemporal action space.

2.2 Production

Suppose a society with one non producible factor of production l (e.g. labor), which is given in each period in amount $\bar{l}$ and two production sectors, each producing a consumption good c_i $(i = 1, 2)$.[1] The consumption good is supposed to be homogenous, thus total consumption is the sum of the production outputs produced by sector 1 and sector 2:

$$c(t) = c_1(t) + c_2(t) \ . \tag{7}$$

The first sector produces the consumption good solely by the means of labor:

$$c_1(t) = l_1(t) \ , \tag{8}$$

where l_1 denotes the amount of labor employed to sector 1. In addition, as an unwanted by-product, sector 1 causes one unit of emissions e for every unit of consumption good produced:

$$e(t) = c_1(t) = l_1(t) \ . \tag{9}$$

Furthermore, consumption can be produced in sector 2, which combines λ units of labor and κ units of a specific capital good k to produce one unit of the consumption good:

$$c_2(t) = \min \left[\frac{l_2(t)}{\lambda}, \frac{k(t)}{\kappa} \right] \ . \tag{10}$$

Analogously to (8), l_2 denotes the labor input employed to sector 2. If the capital stock k is fully employed in every period[2] and efficient labor allocation is supposed, then equation (10) yields:

$$c_2(t) = \frac{l_2(t)}{\lambda} = \frac{k(t)}{\kappa} \ . \tag{11}$$

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[1] This model originates from Faber and Proops (1991), who analyze structural change in a neo-Austrian capital theoretical framework. It has been extended to joint production by Winkler (2002).

[2] If the economy starts with an initial capital stock of $k_1 = 0$ and given the intertemporal welfare W as defined in (1), full employment of the capital stock is also *efficient* as shown by Winkler (2002).

Note that sector 2 does not produce any unwanted joint products. Hence, emissions can be reduced by switching from production sector 1 to production sector 2.

New capital goods are produced by the means of labor. Employing one unit of labor yields one unit of new capital good. Denoting the amount of labor employed to the production of new capital goods by l_3, yields for the investment i:

$$i(t) = l_3(t) \ . \tag{12}$$

A central assumption in this model is that the production of the capital good needs time. This amounts to the assumption that in general the costs and the benefits of investments in environmental quality (in terms of welfare) do not accrue at the same time. A well known example, where costs occur *before* the benefits, is the abatement of CO_2 to slow down the anthropogenic greenhouse effect. While the costs occur today, the (insecure) benefits spread over several decades or even centuries (IPCC 2001b). Hence, the investment i in new capital goods in period t accumulates the existing capital stock k in period $t+1$. Assuming further that the deterioration of capital is proportional to the existing capital stock at the constant and exogenously given rate γ, leads to the following equation of motion for the capital stock k:

$$k(t) = (1 - \gamma)k(t-1) + i(t-1) \ . \tag{13}$$

The exogenously given technical coefficients λ, κ and γ specify the production technology. As welfare is strictly increasing in consumption, in the optimum the labor supply $\bar{l}$ will be used up completely by the three production processes in every period t:

$$\bar{l} = l_1(t) + l_2(t) + l_3(t) \ . \tag{14}$$

Suppose that sector 1 is the status quo in the economy, which is used to its maximal extend. From period $t = 1$ on, the society is aware of the (potential) harmfulness of the jointly produced output. In the following I examine if it is optimal for agent t to invest in the clean production sector 2 for a given set of technical coefficients λ, κ and γ, and a given behavioral pattern as described in definitions 1–3.

3 Ex ante intertemporal optimal consumption and investment

First, I derive the ex ante intertemporal optimal consumption and investment plan. This is the plan agent 1 achieves by maximizing her intertemporal welfare in period $t = 1$ subject to the production possibilities of the economy.

3.1 Intertemporal optimization

Inserting equations (8), (11) and (12) in the labor restriction (14) yields:

$$c_1(t) = e(t) = \bar{l} - \frac{\lambda}{\kappa}k(t) - i(t) \ . \tag{15}$$

Using this equation together with equations (11) and (7), I derive for the total consumption $c(t)$:

$$c(t) = \bar{l} + \frac{1-\lambda}{\kappa}k(t) - i(t) \; . \tag{16}$$

Note that given an initial capital stock $k_1 = k(1)$, the outcome is completely determined by choosing the investment decisions $i(t)$ for all periods $t = 1, \ldots, \tau$. Thus, setting the initial capital stock $k_1 = 0$, the ex ante optimal control problem reads:

$$\max_i \sum_{t=1}^{\tau} D(t) \left[U(c(t)) - S(e(t)) \right] \qquad \text{s.t.} \tag{17}$$

$$
\begin{aligned}
c(t) &= \bar{l} + \frac{1-\lambda}{\kappa}k(t) - i(t) \; , & \forall\, t = 1, \ldots, \tau \; , \\
e(t) &= \bar{l} - \frac{\lambda}{\kappa}k(t) - i(t) \geq 0 \; , & \forall\, t = 1, \ldots, \tau \; , \\
k(t{+}1) &= (1-\gamma)k(t) + i(t) \; , & \forall\, t = 1, \ldots, \tau \; , \\
i(t) &\geq 0 \; , & \forall\, t = 1, \ldots, \tau \; , \\
k_1 &= 0 \; .
\end{aligned}
$$

Introducing shadow prices $p_c(t)$, $p_e(t)$ and $p_k(t)$ for the consumption, the emissions and the capital stock, and a Kuhn-Tucker variable $p_i(t)$ to control for the non-negativity of investment, one obtains the Lagrangian $\mathcal{L}$:[3]

$$
\begin{aligned}
\mathcal{L} =\; & \sum_{t=1}^{\tau} D(t) \left[U(c(t)) - S(e(t)) \right] \\
& + \sum_{t=1}^{\tau} p_c(t) \left[\bar{l} + \frac{1-\lambda}{\kappa}k(t) - i(t) - c(t) \right] \\
& + \sum_{t=1}^{\tau} p_e(t) \left[\bar{l} - \frac{\lambda}{\kappa}k(t) - i(t) - e(t) \right] \\
& + \sum_{t=1}^{\tau} p_k(t{+}1) \left[(1-\gamma)k(t) + i(t) - k(t{+}1) \right] \\
& + \sum_{t=1}^{\tau} p_i(t) i(t) \; .
\end{aligned}
\tag{18}
$$

Hence, the first order conditions for an optimal intertemporal investment plan read:

$$
\begin{aligned}
D(t) U_c(c(t)) - p_c(t) &= 0 \; , & \forall\, t = 1, \ldots, \tau \; , && (19) \\
-D(t) S_e(e(t)) - p_e(t) &= 0 \; , & \forall\, t = 1, \ldots, \tau \; , && (20)
\end{aligned}
$$

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[3] To simplify the exposition, I do not explicitly introduce a Kuhn-Tucker variable to control for the non-negativity of the emissions $e(t)$. This is justified as I am mainly interested in determining the conditions for which there is some investment in the capital intensive production technique at all. Nevertheless, note that due to the linear production processes, a full replacement of process R_1 by process R_2 might occur if investment is optimal.

$$\frac{1-\lambda}{\kappa}p_c(t) - \frac{\lambda}{\kappa}p_e(t) + (1-\gamma)p_k(t{+}1) - p_k(t) = 0 \ , \qquad \forall\, t = 1, \ldots, \tau \ , \qquad (21)$$

$$-p_c(t) - p_e(t) + p_k(t{+}1) + p_i(t) = 0 \ , \qquad \forall\, t = 1, \ldots, \tau \ , \qquad (22)$$

$$p_i(t) \geq 0 \ , \quad p_i(t)i(t) = 0 \ , \qquad \forall\, t = 1, \ldots, \tau \ . \qquad (23)$$

Because of the strict concavity of the Lagrangian (strictly concave objective function and linear restrictions), these necessary conditions are also sufficient if, in addition, the following transversality condition holds:

$$p_k(\tau + 1) = 0 \ . \tag{24}$$

The economic interpretation of the necessary and sufficient conditions is straight forward. Equation (19) claims that for an intertemporal optimal plan the shadow price of consumption equals the present value of the marginal utility of consumption. Analogously, according to equation (20), in the optimum the shadow price of emissions equals the present value of the marginal welfare loss due to environmental damage. Note that the shadow price of emissions $p_e(t)$ is negative in the optimum as emissions decrease welfare.

Equation (21) is a difference equation, which can be solved unambiguously if the transversality condition (24) is taken into account:

$$p_k(t) = \frac{1}{\kappa} \sum_{m=t}^{\tau} D(m)(1-\gamma)^{m-t} \left[(1-\lambda)U_c(c(m)) + \lambda S_e(e(m)) \right] \ . \tag{25}$$

The term in brackets on the right hand side is the net welfare gain of a marginal unit of the capital good in one period. As capital goods are long-lived commodities, the welfare gains of different periods have to be accumulated by taking account of discounting and the depreciation of capital goods. Thus, in the optimum the shadow price of capital equals the present value of the accumulated future welfare gain of a marginal unit of the capital good.

Inserting (19), (20) and (25) in equation (22) I derive the following necessary and sufficient conditions for an intertemporal optimal (ex ante) plan ($t = 1, \ldots, \tau$):

$$D(t) \left[U_c(c(t)) - S_e(e(t)) \right] - p_i(t) = \tag{26}$$
$$\frac{1}{\kappa} \sum_{m=t+1}^{\tau} D(m)(1-\gamma)^{m-t-1} \left[(1-\lambda)U_c(c(m)) + \lambda S_e(e(m)) \right] \ .$$

Equation (26) states that invest in new capital goods in period t (i.e. $p_i(t) = 0$) can only be optimal, if the present welfare loss due to the investment in capital goods (left hand side) equals the net present value of the future possible use of this investment (right hand side).

3.2 Exponential versus hyperbolic intertemporal preferences

First, assume that all agents t have exponential intertemporal preferences as described in equation (3). Inserting (3) in equation (26), one obtains:

$$(1+\rho)^{1-t} \left[U_c(c(t)) - S_e(e(t)) \right] - p_i(t) = \tag{27}$$

$$\frac{1}{\kappa} \sum_{m=t+1}^{\tau} \frac{(1-\gamma)^{m-t-1}}{(1+\rho)^{m-1}} \left[(1-\lambda)U_c(c(m)) + \lambda S_e(e(m)) \right] \ .$$

Given exponential intertemporal preferences, it cannot be optimal not to invest in capital goods in period $t = 1$ but to invest in later periods $t > 1$. This is true, because the relative weights between the welfare loss today and the future benefits remain unaltered by a transition in time, due to the constat rate of time preference, but the time span over which the new capital can produce consumption goods declines. Hence, the welfare loss stays constant (in current values), while the future welfare gain declines the more the later one starts to invest.

Suppose it is *not optimal* to invest in the capital intensive technique in period $t = 1$ (and in all later periods). Then, according to (23), $i(t) = 0$ and $p_i(t) \geq 0$ for all $t = 1, \ldots, \tau$. As a consequence, all labor will be employed in production sector 1 in each and every period t, yielding the constant consumption $\bar{c} = \bar{l}$ and constant emissions $\bar{e} = \bar{l}$. Thus, equation (27) states for period $t = 1$:

$$U_c(\bar{c}) - S_e(\bar{e}) \geq \frac{(1-\lambda)U_c(\bar{c}) - \lambda S_e(\bar{e})}{\kappa} \sum_{m=2}^{\tau} \frac{(1-\gamma)^{m-2}}{(1+\rho)^{m-1}} \ . \tag{28}$$

Using the formula for the geometric series one obtains that investment in the capital intensive production technique is *optimal*, if and only if:

$$\frac{\kappa[U_c(\bar{c}) - S_e(\bar{e})]}{(1-\lambda)U_c(\bar{c}) + \lambda S_e(\bar{e})} < \frac{(1+\rho)^{\tau-1} - (1-\gamma)^{\tau-1}}{(1+\rho)^{\tau-1}(\gamma+\rho)} \ . \tag{29}$$

Second, suppose the agents have hyperbolic preferences. Suppose further that it is *not optimal* to invest in the capital good in all periods t. Then, again all labor will be employed in production sector 1 in each and every period t, yielding $\bar{c} = \bar{l}$ and $\bar{e} = \bar{l}$. Hence, we derive for period t:

$$D(t) \left[U_c(\bar{c}) - S_e(\bar{e}) \right] \geq \frac{(1-\lambda)U_c(\bar{c}) - \lambda S_e(\bar{e})}{\kappa} \sum_{m=t+1}^{\tau} D(m)(1-\gamma)^{m-t-1} \ . \tag{30}$$

Different from the case of exponential intertemporal preferences, now it can be optimal not to invest in period $t = 1$ but in one or more later periods. This is true, because the present value of future benefits increases by a transition in time due to the decreasing instantaneous rates of time preference. Hence, the welfare loss stays constant (in current values), while the future welfare gains decline on the one hand, because the remaining time horizon declines, but rise on the other hand due to the declining rates of time preference. Hence, in the case of hyperbolic intertemporal preferences, investment in the capital good in period t is optimal, if and only if:

$$\frac{\kappa[U_c(\bar{c}) - S_e(\bar{e})]}{(1-\lambda)U_c(\bar{c}) + \lambda S_e(\bar{e})} < \sum_{m=t+1}^{\tau} \frac{D(m)}{D(t)}(1-\gamma)^{m-t-1} \ . \tag{31}$$

If investment is optimal, i.e. equation (31) holds for some t, but equation (31) does not hold for $t = 1$, then it is optimal for the hyperbolically discounting agent to postpone

the investment in the clean technology into the future. I term this delay *ex ante procrastination* to distinguish it from *ex post procrastination* described in the next section. The following proposition summarizes this result.

Proposition 1 (Ex ante procrastination)
Given the maximization problem (17) and hyperbolically discounting agents as described in (5) and (6), the ex ante intertemporal optimal plan can exhibit ex ante procrastination, *i.e. investment in the clean production sector is ex ante optimal in the long run but not in the first period.*

Note that within this model ex ante procrastination cannot be optimal, if the agents discount exponentially. Hence, this is a special feature of hyperbolic preferences.

4 Ex post implemented consumption and investment

As described in section 2, the agents might not stick to the ex ante optimal plan, if they reevaluate it in later periods, because of the non-stationarity of their hyperbolic preferences. Hence, depending on the behavior pattern assumed, the ex post actually implemented plan can differ from the ex ante intertemporal optimal plan.

4.1 Hyperbolic committed discounting

As described in definition 1, the hyperbolic committed agent 1 derives the ex ante intertemporal optimal investment and consumption plan, implements the investment $i(1)$ for the first period and mandatory commits her successors to the future investments as suggested by the ex ante plan. As a consequence, even if the succeeding agents want to depart from the ex ante plan due to reevaluations in later periods, they have to stick to it. Obviously, in this case the ex post implemented plan is identical to the ex ante optimal plan.

According to the analysis exposed in the former section, investment in the clean production technique is ex ante optimal, if equation (31) holds for some t. Suppose the t which maximizes the right hand side of (31) is t'. Hence, the hyperbolic committed agent t' will invest in the capital intensive technique, if (31) holds for t', as the following proposition states.

Proposition 2 (Hyperbolic committed discounting)
Suppose an agent with hyperbolic preferences as described by (5) and (6), and a hyperbolic committed behavior pattern according to definition 1. Given the maximization problem (17), the agent will invest into the clean production sector, if and only if:

$$\frac{\kappa[U_c(\bar{c}) - S_e(\bar{e})]}{(1-\lambda)U_c(\bar{c}) + \lambda S_e(\bar{e})} < \max_t \left[\sum_{m=t+1}^{\tau} \frac{D(m)}{D(t)} (1-\gamma)^{m-t-1} \right] \ .$$

Note that the ex post implemented plan of a hyperbolic committed agent exhibits ex ante procrastination, if and only if the ex ante optimal plan does.

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4.2 Hyperbolic myopic discounting

The hyperbolic myopic agents are not aware of the non-stationarity of their preferences. Hence, agent 1 will maximize her intertemporal welfare in period $t = 1$, derive the ex ante optimal plan as described in section 3 and implement the putative optimal investment $i(1)$. She is not aware that $i(1)$ is only optimal, if her successors stick to the future investments as derived by the ex ante optimal plan. In period $t = 2$ agent 2 will reevaluate the optimal plan. In general, the optimal results derived by the reevaluation do not coincide with the ex ante optimal plan, because the relative weights of welfare gains and losses between different periods have changed due to the declining instantaneous rates of time preference. Again, she will implement the alleged optimal investment $i(2)$. In period $t = 3$ agent 3 reevaluates the optimal plan again and will alter the previously derived decision and so on. The behavior of the hyperbolic myopic agents are irrational as they do not learn from the permanent reevaluation experiences of their predecessors.

Let's turn to the question under which circumstances the hyperbolic myopic agent will invest into the clean production technique. As we have seen already, if the ex ante optimal plan suggests investment in period $t = 1$, she will invest in exactly the proposed amount. Now suppose, she faces ex ante procrastination, i.e. the ex ante optimal plan suggests to invest in the clean production technology not earlier than in period $t' > 1$. Hence, agent 1 will not invest in period $t = 1$. But what about agent t'? As shown in the appendix, she will invest in period $t = t'$, as suggested by the ex ante optimal plan, if and only if:

$$\frac{\kappa[U_c(\bar{c}) - S_e(\bar{e})]}{(1 - \lambda)U_c(\bar{c}) + \lambda S_e(\bar{e})} < \sum_{n=t'+1}^{\tau} D(n - t' + 1)(1 - \gamma)^{n-t'-1} \ . \tag{32}$$

But if (32) holds, investment would have been already optimal in period $t = 1$, because in the condition for optimal investment in $t = 1$, the left hand side is the same, but the right hand side (RHS) is even larger as the sum contains the same terms plus some additional positive addends:

$$\sum_{n=t'+1}^{\tau} D(n - t' + 1)(1 - \gamma)^{n-t'-1} = \tag{33}$$

$$\underbrace{\sum_{m=2}^{\tau-t'+1} D(m)(1 - \gamma)^{m-2}}_{\text{RHS for } t=t'} < \underbrace{\sum_{m=2}^{\tau} D(m)(1 - \gamma)^{m-2}}_{\text{RHS for } t=1} \ .$$

This contradicts the assumption that the ex ante optimal plan exhibits ex ante procrastination. As a consequence, the hyperbolic myopic agents will never invest in the clean production sector, if they do not invest in the first period. I call this (infinite) delay caused by the time-inconsistency problem *ex post procrastination*. The following proposition summarizes this result.

Proposition 3 (Hyperbolic myopic discounting)
Suppose agents with hyperbolic preferences as described by (5) and (6), and a hyperbolic myopic behavior pattern according to definition 2. Given the maximization problem (17), agent 1 will invest into the clean production technique, if and only if:

$$\frac{\kappa[U_c(\bar{c}) - S_e(\bar{e})]}{(1 - \lambda)U_c(\bar{c}) + \lambda S_e(\bar{e})} < \sum_{m=2}^{\tau} D(m)(1 - \gamma)^{m-2} \ .$$

Furthermore, if agent 1 does not invest, nor do all succeeding agents.

A direct consequence of proposition 3 is that in the case of hyperbolic myopic agents, ex ante procrastination implies ex post procrastination. That means, if the ex ante optimal plan exhibits ex ante procrastination, the hyperbolic myopic agents will *never* invest in the capital good, although agent 1 thought that her successors would, according to the ex ante optimal plan.

4.3 Hyperbolic non-cooperative discounting

As stated in definition 3, the hyperbolic non-cooperative agents are aware of the time-inconsistency problem but have no possibility to mandatory commit their successors to the ex ante plan. In this case the best they can do, is playing a non-cooperative sequential game against all other agents. Then, the time consistent consumption and investment plan is the subgame perfect Nash equilibrium of this game.

As known from game theory, the Nash equilibrium is the mutual best-response of all agents t, given the investment decisions of all other agents. As shown in the appendix, maximizing each agents intertemporal welfare, given the investment decisions of all other agents, leads to the following system of nonlinear equations ($t = 1, \ldots, \tau$), whose unique solution describes the investment plans of all agents in the subgame perfect Nash equilibrium:

$$U_c(c(t)) - S_e(e(t)) - p_i(t) = \tag{34}$$
$$\frac{1}{\kappa} \sum_{n=t+1}^{\tau} D(n-t+1)(1-\gamma)^{n-t-1} \left[(1-\lambda)U_c(c(n)) + \lambda S_e(e(n))\right] \ .$$

Suppose that in the subgame perfect Nash equilibrium no agent t invests in the capital intensive production technique, i.e. $i(t) = 0$, $p_i(t) \geq 0$ ($t = 1, \ldots, \tau$). Then, in each and every period the consumption $\bar{c}$ and the emissions $\bar{e}$ will be implemented. Hence, for all agents t the following inequality holds:

$$\frac{\kappa U_c(\bar{c}) - S_e(\bar{e})}{[(1 - \lambda)U_c(\bar{c}) + \lambda S_e(\bar{e})]} \geq \sum_{n+t+1}^{\tau} D(n-t+1)(1 - \gamma)^{n-t-1} \ . \tag{35}$$

Turning it the other way round, agent t' will depart, if and only if:

$$\frac{\kappa[U_c(\bar{c}) - S_e(\bar{e})]}{(1 - \lambda)U_c(\bar{c}) + \lambda S_e(\bar{e})} < \sum_{n=t'+1}^{\tau} D(n-t'+1)(1 - \gamma)^{n-t'-1} \ . \tag{36}$$

This condition is identical to condition (32) for an ex post implemented investment of the hyperbolic myopic agents. Exploiting the same line of argument, it follows that in the subgame perfect Nash equilibrium no agent t invests in sector 2, if agent 1 does not invest. As a consequence, also in the case of hyperbolic non-cooperative agents, ex ante procrastination implies (infinite) ex post procrastination. The following proposition summarizes this result.

Proposition 4 (Hyperbolic non-cooperative discounting)
Suppose agents with hyperbolic preferences as described by (5) and (6), and a hyperbolic non-cooperative behavior pattern according to definition 3. Given the maximization problem (17), agent 1 will invest into the clean production technique, if and only if:

$$\frac{\kappa[U_c(\bar{c}) - S_e(\bar{e})]}{(1 - \lambda)U_c(\bar{c}) + \lambda S_e(\bar{e})} < \sum_{m=2}^{\tau} D(m)(1 - \gamma)^{m-2} \ .$$

Furthermore, if agent 1 does not invest, nor do all succeeding agents.

Note that, although the necessary and sufficient condition for an ex post implemented investment in the clean production technique is identical for both the hyperbolic myopic and the hyperbolic non-cooperative agents, the optimal dynamic path in case of an investment will in general differ for the various behavior patterns (see section 6). Hence, propositions 3 and 4 do not imply that the optimization calculus for both behavior patterns is identical.[4]

5 Welfare analysis and intertemporal equity

Let's turn to the ex post derived welfare, which depends of the different behavior patterns described by definitions 1–3. Agent t's ex post derived welfare is given by (1), when inserting the actually implemented ex post plan. In the following I am concerned about intertemporal equity. As equity is not an economic but an ethical concept, I am obviously in an insecure position when analyzing intertemporal equity. As a consequence, I apply a very weak definition of intertemporal equity (in fact, I give a very *strong* definition of *inequity*): A behavioral regime is called *inequitable*, if the decision of agent t is determined by the dictatorship of another agent $t' \neq t$.[5]

First, suppose the case of exponential discounting, i.e. $D(t) = D^{exp}(t)$ as given in (3). Then, the intertemporal distribution of welfare of the ex ante optimal plan among the agents living in different periods t is Pareto optimal insofar, as it is not possible to increase the intertemporal welfare of one agent without decreasing the welfare of another agent. In addition, the distribution is optimal for every agent t, as no one could improve its own welfare (even at the cost of others) by departing from the ex ante optimal plan. Apparently, the exponential discounting regime is very favorable, as it is efficient and it is not inequitable in the sense defined above.

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[4] In fact, they are quite different as can be seen from their derivation in the appendix.
[5] This definition is very similar to the *non-dictatorship*-property in the Arrow's paradox (Arrow 1951).

Second, suppose the hyperbolic committed discounting scheme. Then, as shown in section 4.1, the ex post implemented plan is identical to the ex ante optimal plan. Again, the intertemporal distribution of welfare among the agents is Pareto optimal, as it is not possible to increase the intertemporal welfare of any agent t $(t = 2, \ldots, \tau)$ without decreasing agent 1's welfare. But in general the distribution is suboptimal for all agents except agent 1. Furthermore, the hyperbolic committed discounting regime is inequitable as the individual optimal decisions of all agents t $(t = 2, \ldots, \tau)$ are overruled by the commitment of agent 1 in period 1.

Third, assume that all agents discount according to the hyperbolic myopic or hyperbolic non-cooperative behavior pattern. Obviously, these discounting schemes are not inequitable in the strict sense defined above, as every agent is able to implement its own investment decision. But the ex post implemented plan may be Pareto inefficient, depending on the set of exogenous given parameters and the functional form of the discount and the utility function as shown in the appendix. In this case, one faces the dilemma that the ex post implemented plan is *either* efficient *or* non-dictatorial. The following proposition summarizes this result.

Proposition 5 (Efficiency-equity trade-off)
Suppose agents with hyperbolic preferences as given in (5) and (6). Then, the ex post implemented plan is efficient but dictatorial in the hyperbolic committed regime, and non-dictatorial but may be inefficient in the hyperbolic myopic and the hyperbolic non-cooperative regime.
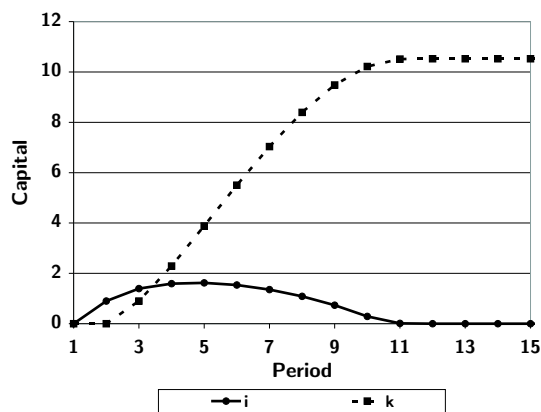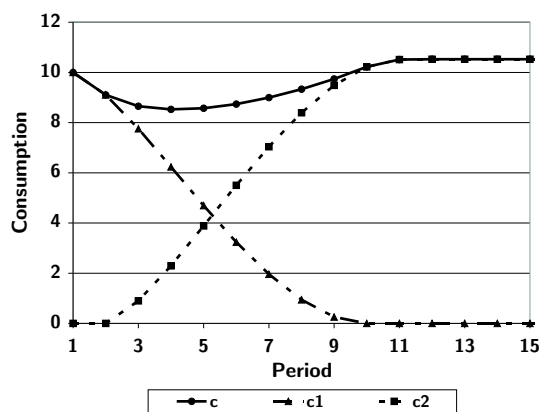
6 Numerical simulation

In this section the results obtained in sections 3 and 4 are illustrated by numerical optimizations. All numerical optimizations were derived with the advanced optimal control software package MUSCOD-II (Diehl et al. 2001), which exploits the multiple shooting state discretization (Leineweber et al. 2003). The numerical optimization parameters are given in the appendix. Note that there is no deterioration of the capital stock in the numerical examples ($\gamma = 0$).
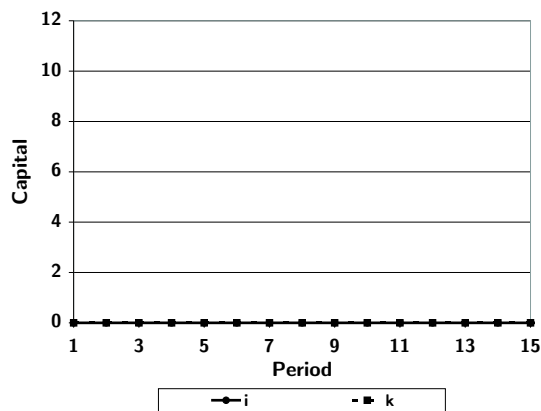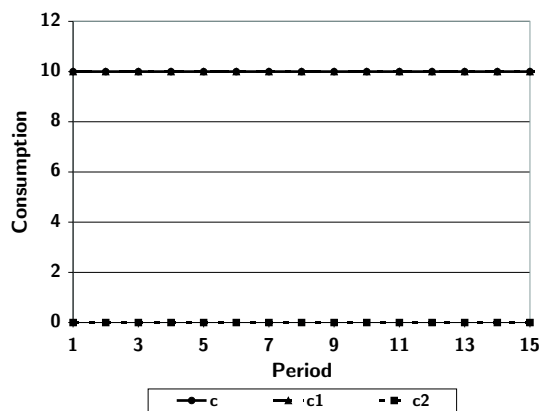
Figure 2 shows a numerical calculated ex post implemented plan for the three different behavior patterns, given ex ante procrastination. As expected from propositions 3 and 4, the ex post implemented plans for hyperbolic myopic and hyperbolic non-cooperative agents also exhibit ex post procrastination, i.e. there is no investment in the clean production sector in all periods. As a consequence, the status quo consumption and emissions are obtained in every period. However, the hyperbolic committed agents start with the status quo consumption of $c = 10$ in period 1 as agent 1 does not invest in this period. Between period 2 and 9 the consumption is below 10 due to the investments in the capital stock.

Figure 3 shows a numerical calculated ex post implemented plan, where investment is undertaken in all three behavioral regimes. Note that given the time horizon $\tau = 25$ (in figure 2 and 3 only the first 15 periods are shown for a more convenient presentation) only the hyperbolic committed agents realize a full replacement of the polluting production

a) hyperbolic committed discounting

b) hyperbolic myopic discounting

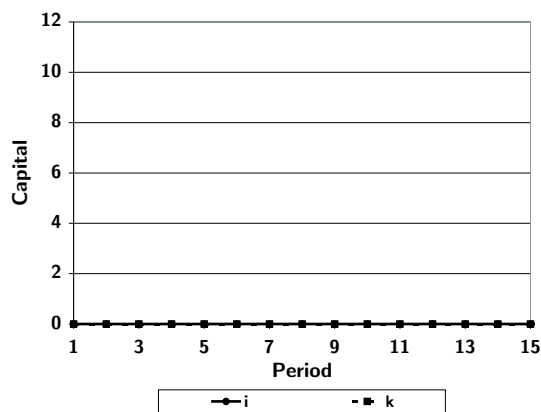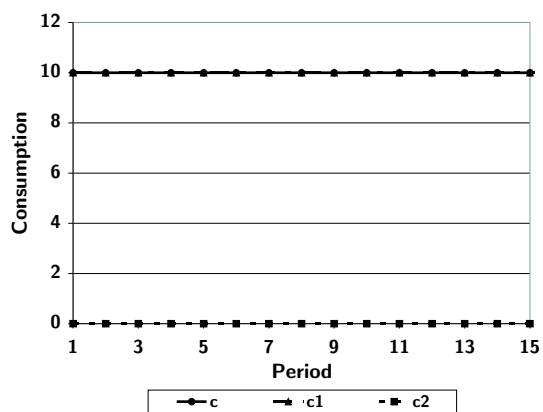## c) hyperbolic non-cooperative discounting

Figure 2: Consumption (left), and capital and investments (right) for the hyperbolic committed, the hyperbolic myopic and the hyperbolic non-cooperative representative agents in the case of ex ante procrastination.

15

sector 1 by the clean production sector 2. Note further that the investment path is monotonic for the hyperbolic myopic and the hyperbolic non-cooperative agents while it is non-monotonic for the hyperbolic committed agents. This is because the relative weights between costs today and benefits in the future change due to the hyperbolic discount function of the hyperbolic committed agents, while they don't (at least in the ex post implemented plan) for the hyperbolic myopic and hyperbolic non-cooperative agents.

In table 1 the intertemporal welfare of the agents in all periods are given for the different numerical examples. In the case of ex ante procrastination (figure 2), the agents in all periods are better off in the hyperbolic committed than in the hyperbolic myopic and hyperbolic non-cooperative regime. In fact, in this example the inefficiency of the ex post implemented plan of the hyperbolic myopic and the hyperbolic non-cooperative agents is obvious. Note that it is not necessarily the case that all agents are better off in the hyperbolic committed scenario even if the ex ante intertemporal optimal plan exhibits ex ante procrastination.

In the scenario, where we observe investment in all three discounting regimes (figure 3), again the hyperbolic myopic agents have a lower intertemporal welfare than the hyperbolic committed representative consumer, which clearly indicates an inefficient intertemporal allocation. Indeed, in the hyperbolic non-cooperative regime the agents 2 to 6 are better off than the corresponding agents in the hyperbolic committed regime. Nevertheless, this outcome is inefficient as a Pareto improvement can easily be constructed.[6]
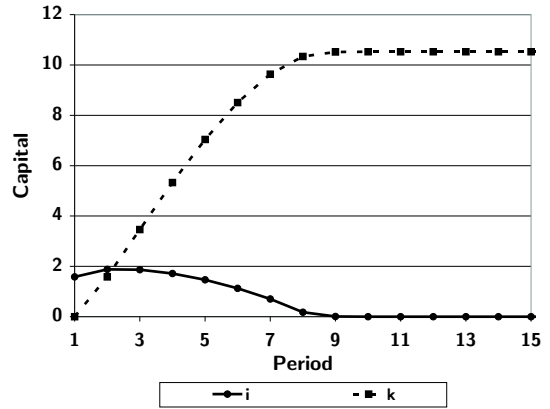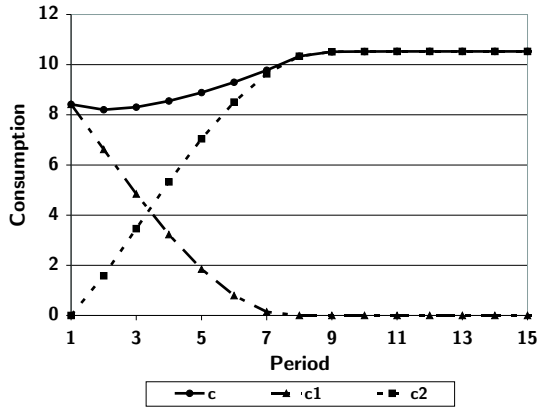
7 Conclusion

I have analyzed optimal intertemporal structural change from a polluting to a non polluting production sector for hyperbolically discounting agents. Because of the non-stationarity of hyperbolic preferences, the ex post observed outcome crucially depends on additional behavioral constraints. As prime examples I have discussed the hyperbolic myopic, the hyperbolic committed and the hyperbolic non-cooperative agents, which correspond to the corners of the two dimensional behavioral manifold spanned by the possibilities of commitment to future actions and the degree of foresight of one's own time inconsistent behavior. The well known result that hyperbolic agents tend to postpone costs has been extended by a distinction between ex ante procrastination, a direct consequence of declining discount rates, and ex post procrastination, a phenomenon which amounts to the problem of time inconsistent behavior in the absence of commitment possibilities.[7]

In the model framework analyzed I have shown that, if the agents exhibit hyperbolic myopic or hyperbolic non-cooperative behavior, ex ante procrastination implies an infinite ex post procrastination, i.e. there is no investment in the clean production technique

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[6] An example for a Pareto superior investment plan is $i(1) = 2.5$, $i(2) = 2.1$, $i(3) = 1.8$, $i(4) = 1.5$, $i(5) = 1.2$, $i(6) = 0.8$, $i(7) = 0.3$, $i(8) = \ldots = i(25) = 0$.

[7] Note that ex ante procrastination is similar to the result in the real options theory, where it can be optimal to postpone an investment if its future benefits are uncertain and the uncertainty reduces as time increases even with exponential discounting (e.g. Dixit and Pindyck 1994).

a) hyperbolic committed discounting

b) hyperbolic myopic discounting

c) hyperbolic non-cooperative discounting

Figure 3: Consumption (left), and capital and investments (right) for the hyperbolic committed, the hyperbolic myopic and the hyperbolic non-cooperative agents in the absence of ex ante procrastination.
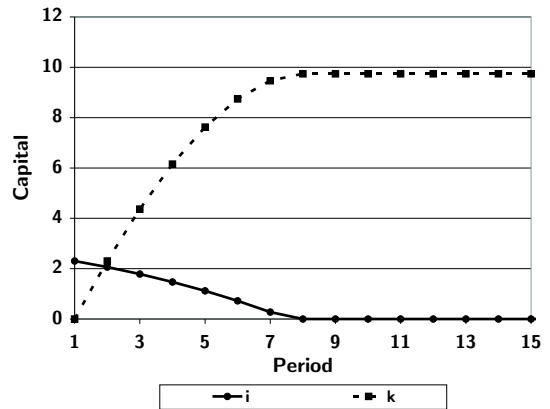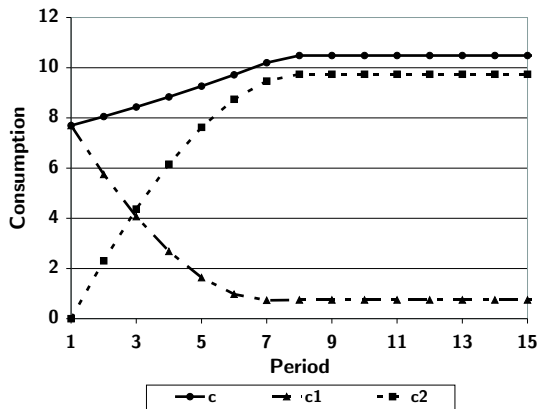
Agent	Figure 2			Figure 3		
	committed	myopic	non-coop.	committed	myopic	non-coop.
1	27,99	27,91	27,91	41,87	41,82	41,85
2	27,15	27,11	27,11	40,55	40,51	40,62
3	26,37	26,29	26,29	39,25	39,17	39,35
4	25,65	25,46	25,46	37,94	37,80	38,02
5	24,93	24,63	24,63	36,59	36,40	36,65
6	24,20	23,78	23,78	35,20	34,97	35,22
7	23,44	22,91	22,91	33,76	33,50	33,74
8	22,66	22,03	22,03	32,26	31,97	32,20
9	21,84	21,14	21,14	30,69	30,40	30,62
10	20,97	20,23	20,23	29,10	28,82	29,03
11	20,03	19,30	19,30	27,49	27,23	27,43
12	19,05	18,35	18,35	25,87	25,62	25,81
13	18,04	17,38	17,38	24,23	24,00	24,18
14	17,01	16,39	16,39	22,58	22,36	22,53
15	15,95	15,37	15,37	20,91	20,71	20,86
16	14,87	14,32	14,32	19,21	19,03	19,17
17	13,74	13,24	13,24	17,50	17,33	17,46
18	12,58	12,12	12,12	15,76	15,61	15,72
19	11,38	10,96	10,96	13,99	13,86	13,96
20	10,12	9,75	9,75	12,19	12,07	12,16
21	8,80	8,48	8,48	10,35	10,25	10,32
22	7,40	7,12	7,12	8,46	8,38	8,44
23	5,89	5,67	5,67	6,52	6,45	6,50
24	4,24	4,08	4,08	4,49	4,45	4,48
25	2,35	2,27	2,27	2,35	2,33	2,35

Table 1: Intertemporal welfare of all agents in the different numerical examples.

in all periods, although this has been optimal from an ex ante point of view. This infinite ex post delay is the result of the assumption of a flow pollutant. If not the emissions themselves decrease welfare, but the emissions accumulate a stock of pollutant, which has a welfare deceasing property, only finite ex post procrastination would result. Nevertheless, the qualitative difference between ex ante and ex post procrastination would persist.

Furthermore, I have shown that ex post procrastination may lead to a Pareto inefficient intertemporal outcome. Note that the inefficiency hinges upon the set of exogenously given parameters and the functional forms of D, U and S. Although the hyperbolic committed behavioral regime produces always a Pareto optimal outcome, it has the unfavorable property of a dictatorship of the present agent over all future agents. One might get over this as long as time-inconsistency is a pure *intrapersonal* problem, i.e. the different agents t are one physical person at different times t. But, once discussing *inter-*

personal or *intergenerational* problems, i.e. the different agents t are different physical persons living at different times t, a dictatorship of the first generation over all others is out of question, either on ethical grounds or simply by the mere impossibility to commit future generations to actions which are suboptimal in their view. Hence, there is a potential trade-off between intertemporal efficiency and equity. The solution of this problem is open to future research.[8]

Nevertheless, I believe that the differentiation between ex ante and ex post procrastination contributes to the explanation of the observed behavior of decision makers in long-term projects. As an example think of the reduction of greenhouse gas emissions according to the Kyoto protocol (United Nations Third Conference of the Parties of the Framework Convention on Climate Change 1997). Suppose the commitment of the US government signed in 1997 exhibited ex ante procrastination. As the 1997 government had no means to enforce future governments' cooperation and/or was unaware of their time-inconsistent preferences, today's and future governments are not willing to ratify the Kyoto protocol, because of ex post procrastination.

Appendix

A.1 Hyperbolic myopic discounting

The intertemporal optimization problem the hyperbolic myopic agent has to solve in every period t is similar to the ex ante intertemporal optimization problem (17). Nevertheless, the remaining time horizon declines with increasing t, i.e. the time span, over which capital goods can be used, also declines. The initial capital stock k_t is determined by the investment carried out in the former periods:

$$\max_i \sum_{m=t}^{\tau} D(m-t+1) \left[U(c(m)) - S(e(m)) \right] \qquad \text{s.t.} \tag{A.1}$$

$$c(m) = \bar{l} + \frac{1-\lambda}{\kappa} k(m) - i(m) , \qquad \forall\, m = t, \ldots, \tau ,$$

$$e(m) = \bar{l} - \frac{\lambda}{\kappa} k(m) - i(m) \geq 0 , \qquad \forall\, m = t, \ldots, \tau ,$$

$$k(m+1) = (1-\gamma)k(m) + i(m) , \qquad \forall\, m = t, \ldots, \tau ,$$

$$i(m) \geq 0 , \qquad \forall\, m = t, \ldots, \tau ,$$

$$k_t = \sum_{n=1}^{t-1} (1-\gamma)^{t-n-1} i(n) .$$

Analogously to the calculation in section (3.1), we derive the following system of equations for every period t, which determines the investment in period t and the putative optimal investments in the later periods ($m = t, \ldots, \tau$):

$$D(m-t+1) \left[U_c(c(m)) - S_e(e(m)) \right] - p_i(m) = \tag{A.2}$$

---

[8] Cropper and Laibson (1996) suggest to Pareto improve the outcome by subsidizing the interest rate. Their crucial assumption is that the effect of implemented policies occur time-lagged, which is identical to a commitment for the next period. Hence, the fundamental dilemma remains.

$$\frac{1}{\kappa} \sum_{n=m+1}^{\tau} D(n-m+1)(1-\gamma)^{n-m-1} \left[(1-\lambda)U_c(c(n)) + \lambda S_e(e(n)) \right] \ .$$

Hence, following the same argument as described in detail in section 3.2, investment in period $t = t'$ is assessed to be optimal by the reevaluation in period t' and thus will be carried out, if and only if:

$$\frac{\kappa[U_c(\bar{c}) - S_e(\bar{e})]}{(1-\lambda)U_c(\bar{c}) + \lambda S_e(\bar{e})} < \sum_{n=t'+1}^{\tau} D(n-t'+1)(1-\gamma)^{n-t'-1} \ . \tag{A.3}$$

A.2 Hyperbolic non-cooperative discounting

In the subgame perfect Nash equilibrium every player t plays the best investment strategy, given the investment strategies of all other players. Hence, each player t plays the solution of the following optimization problem:

$$\max_{i(t)} \sum_{m=t}^{\tau} D(m-t+1) \left[U(c(m)) - S(e(m)) \right] \qquad \text{s.t.} \tag{A.4}$$

$$c(m) = \bar{l} + \frac{1-\lambda}{\kappa} k(m) - i(m) \ , \qquad \forall\, m = t, \ldots, \tau \ ,$$

$$e(m) = \bar{l} - \frac{\lambda}{\kappa} k(m) - i(m) \geq 0 \ , \qquad \forall\, m = t, \ldots, \tau \ ,$$

$$k(m+1) = (1-\gamma)k(m) + i(m) \ , \qquad \forall\, m = t, \ldots, \tau \ ,$$

$$i(t) \geq 0 \ , \qquad \forall\, m = t, \ldots, \tau \ ,$$

$$i(m) = i_m \ , \qquad \forall\, m = 1, \ldots, \tau \ , \qquad m \neq t \ ,$$

$$k_t = \sum_{n=1}^{t-1} (1-\gamma)^{t-n-1} i_n \ .$$

Analogously to the calculation in section (3.1), we derive the following necessary and sufficient condition for an optimal investment of player t:

$$U_c(c(t)) - S_e(e(t)) - p_i(t) = \tag{A.5}$$
$$\frac{1}{\kappa} \sum_{n=t+1}^{\tau} D(n-t+1)(1-\gamma)^{n-t-1} \left[(1-\lambda)U_c(c(n)) + \lambda S_e(e(n)) \right] \ .$$

The subgame perfect Nash equilibrium can be determined by backward induction. This leads to the following system of equations, whose solution describes the investment decisions of all players in the Nash equilibrium:

$$U_c(c(1)) - S_e(e(1)) - p_i(1) =$$
$$\frac{1}{\kappa} \sum_{n=2}^{\tau} D(n)(1-\gamma)^{n-2} \left[(1-\lambda)U_c(c(n)) + \lambda S_e(e(n)) \right] \ ,$$

$$U_c(c(2)) - S_e(e(2)) - p_i(2) =$$
$$\frac{1}{\kappa} \sum_{n=3}^{\tau} D(n-1)(1-\gamma)^{n-3} \left[(1-\lambda)U_c(c(n)) + \lambda S_e(e(n)) \right] \ ,$$

$$\vdots \qquad\qquad\qquad (A.6)$$

$$U_c(c(\tau-1)) - S_e(e(\tau-1)) - p_i(\tau-1) =$$
$$\frac{1}{\kappa} D(2) \left[(1-\lambda)U_c(c(\tau)) + \lambda S_e(e(\tau)) \right] \ ,$$
$$U_c(c(\tau)) - S_e(e(\tau)) - p_i(\tau) = 0 \ .$$

Note that due to the curvature properties of the Lagrangian of each player t a unique subgame perfect Nash equilibrium exists.

A.3 Proof of proposition 5

The task is to show that the ex post implemented plan given hyperbolic myopic or hyperbolic non-cooperative agents might be inefficient. In the following I construct an example for both cases.

Suppose $\tau = 3$ and that the ex ante optimal plan exhibits ex ante procrastination.[9] Hence, in the ex post implemented plan no investment is undertaken in all periods for both the hyperbolic myopic and the hyperbolic non-cooperative agents. Introducing the following abbreviations

$$B = \frac{(1-\lambda)U_c(\bar{c}) + \lambda S_e(\bar{e})}{\kappa} \ , \quad C = U_c(\bar{c}) - S_e(\bar{e}) \ , \qquad (A.7)$$

from propositions 3 and 4 follows:

$$C \ \geq \ B\left[D(2) + D(3)(1-\gamma) \right] \ , \qquad (A.8)$$
$$C \cdot D(2) \ < \ B \cdot D(3) \ . \qquad (A.9)$$

Starting from this intertemporal investment plan, I show that depending on the exogenously given parameters and the functional form of D, U and S a Pareto improvement might be possible. Suppose agent 1 invests a marginal unit Δi_1. According to (A.8), this investment will decrease her welfare while it increases the welfare of agent 2 and agent 3. Suppose further that agent 2 also invests a marginal unit Δi_2. According to (A.9) this investment is welfare increasing for agent 1 and agent 3 but welfare decreasing for agent 2. If there are positive investments Δi_1 and Δi_2 such that both agent 1's and agent 2's welfare increase compared to the case of no investment in all periods, then no investment is Pareto inefficient. Note that an investment Δi_3 in period $t = 3$ is always inefficient as the world ends after period 3.

Agent 1's welfare change ΔW_1 due to the investments Δi_1 and Δi_2 reads

$$\Delta W_1 \ = \ -\Delta i_1 \cdot C + \Delta i_1 \left[D(2) + D(3)(1-\gamma) \right] \qquad (A.10)$$
$$-\Delta i_2 \cdot C \cdot D(2) + \Delta i_2 \cdot B \cdot D(3) \ .$$

[9] Note that both assumptions are not necessary and are just chosen to simplify the exposition.

Note that the first line is negative according to (A.8) and the second line is positive due to (A.9). Hence, ΔW_1 is the bigger the higher the marginal investment Δi_2. But agent 2 can only invest in such amount that she is not worse off than in the case of no investment:

$$\Delta W_2 = \Delta i_1 \cdot B\left[1 + D(2)(1-\gamma)\right] - \Delta i_2 \cdot C + \Delta i_2 \cdot B \cdot D(3) = 0 , \qquad \text{(A.11)}$$

$$\Rightarrow \Delta i_2 = \Delta i_1 \frac{B\left[1 + D(2)(1-\gamma)\right]}{C - B \cdot D(2)} > 0 . \qquad \text{(A.12)}$$

Note that $\Delta i_2 > 0$ according to (A.9). Inserting (A.12) in (A.10) and rearranging terms yields:

$$
\begin{aligned}
\frac{\Delta W_1}{\Delta i_1} = {} & -C + B\left[D(2) + D(3)(1-\gamma)\right] \\
& -C \cdot D(2)\frac{B\left[1 + D(2)(1-\gamma)\right]}{C - B \cdot D(2)} + B \cdot D(3)\frac{B\left[1 + D(2)(1-\gamma)\right]}{C - B \cdot D(2)} .
\end{aligned}
\qquad \text{(A.13)}
$$

Depending on the exogenously given parameters and the functional form of D, U and S, (A.13) may be positive or negative. As an example, suppose that

$$\lambda = \kappa = \gamma = 1, \quad U(\bar{c}) = 1.5, \quad S(\bar{e}) = 1 , \qquad \text{(A.14)}$$

which implies $C = 0.5$ and $B = 1$. On the one hand, for $D(2) = 0.25$ and $D(3) = 0.24$ one obtains $\Delta W_1/\Delta i_1 > 0$, indicating that no investment in all periods *is not* Pareto efficient. On the other hand, from $D(2) = 0.25$ and $D(3) = 0.15$ follows that $\Delta W_1/\Delta i_1 < 0$. Hence, in this case no investment in all periods *is* Pareto efficient.

A.4 Parameter values for the numerical optimization

The following welfare function was used for the numerical optimizations:

$$V(c(t), e(t)) = U(c(t)) - S(e(t)) = \ln(c(t)) - \sigma e(t) . \qquad \text{(A.15)}$$

The applied discount function reads:

$$D(t) = (1 + \rho)^{-\frac{\ln(t)}{\ln(2)}} , \qquad \text{(A.16)}$$

where $\rho = \rho(1)$ denotes the discount rate in the first period. In addition, the following parameters were used for the different scenarios:

Figure	τ	$\bar{l}$	λ	κ	γ	σ	ρ
2	25	10	0.95	1	0	0.0035	0.25
3	25	10	0.95	1	0	0.0025	0.1

Although the optimization has been calculated for $\tau = 25$ in figure 2 and 3 only 15 periods are shown for a more convenient presentation.

References

ACKERLOF, G. (1991): 'Procrastination and obedience', *American Economic Review*, **81:** 1–19.

AHLBRECHT, M. AND M. WEBER (1995): 'Hyperbolic discounting models in prescriptive theory of intertemporal choice', *Zeitschrift für Wirtschafts- und Sozialwissenschaften*, **115:** 535–68.

ARROW, K. J. (1951): *Social Choice and Individual Values.* Wiley, New York.

AZFAR, O. (1999): 'Rationalizing hyperbolic discounting', *Journal of Economic Behaviour & Organization*, **38:** 245–52.

BROCAS, I. AND J. D. CARRILLO (2001): 'Rush and procrastination under hyperbolic discounting and independent activities', *Journal of Risk and Uncertainty*, **22:** 141–44.

CHICHILNISKY, G. (1996): 'An axiomatic approach to sustainable development', *Social Choice and Welfare*, **13:** 231–57.

CROPPER, M. AND D. LAIBSON (1996): 'The implications of hyperbolic discounting for project evaluation'. In PORTNEY, P. R. AND J. P. WEYANT (Eds.): *Discounting and Intergenerational Equity.* Resources for the Future, Washington DC.

DEBREU, G. (1954): 'Representation of a preference ordering by a numerical function'. In THRALL, R. M., DAVIS, R. L. AND C. H. COOMBS (Eds.): *Decision Process.* John Wiley, New York. Pages 159–65.

DIEHL, M., LEINEWEBER, D. B. AND A. A. S. SCHÄFER (2001): 'Muscod-II users' manual'. *Preprint 2001-25*, Interdisciplinary Center for Scientific Computing, University of Heidelberg.

DIXIT, A. K. AND R. S. PINDYCK (1994): *Investmnet under Uncertainty.* Princteon University Press, Princeton.

FABER, M. AND J. L. R. PROOPS (1991): 'The innovation of techniques and the time horizon: A neo-austrian approach', *Structural Change and Economic Dynamics*, **2:** 143–58.

FREDERICK, S., LOEWENSTEIN, G. AND T. O'DONOGHUE (2002): 'Time discounting and time preference: A critical review', *Journal of Economic Literature*, **40:** 351–401.

GINTIS, H. (2000): 'Beyond homo economicus: evidence from experimental economics', *Ecological Economics*, **35:** 311–22.

HARRIS, C. AND D. LAIBSON (2001): 'Dynamic choices of hyperbolic consumers', *Econometrica*, **69:** 935–57.

IPCC (2001a): *Climate Change 2001. Mitigation.* Cambridge University Press, Cambridge.

IPCC (2001b): *Climate Change 2001. The Scientific Basis.* Cambridge University Press, Cambridge.

KOOPMANS, T. C. (1960): 'Stationary ordinal utility and impatience', *Econometrica*, **28:** 287–309.

LAIBSON, D. (1997): 'Golden eggs and hyperbolic discounting', *Quarterly Journal of Economics*, **62:** 443–79.

LAIBSON, D. (1998): 'Life-cycle consumption and hyperbolic discount functions', *European Economic Review*, **42:** 861–71.

LEINEWEBER, D. B., BAUER, I., BOCK, H. G. AND J. P. SCHLÖDER (2003): 'An efficient multiple shooting based reduced sqp strategy for large-scale dynamic process optimization – part i: theoretical aspects', *Computers & Chemical Engineering*, **27:** 157–66.

LI, C. Z. AND K. G. LOFGREN (2000): 'Renewable resources and economic sustainability: a dynamic analysis with heterogenous time preferences', *Journal of Environmental Economics and Management*, **40:** 236–50.

LIND, R. C. (Ed.) (1982): *Discounting for Time and Risk in Energy Policy.* John Hopkins University Press, Baltimore.

O'DONOGHUE, T. AND M. RABIN (1999): 'Doing it now or later', *American Economic Review*, **89:** 103–24.

PHELPS, E. S. AND R. A. POLLAK (1968): 'On second-best national saving and game-equilibrium growth', *Review of Economic Studies*, **35:** 185–99.

PORTNEY, P. R. AND J. P. WEYANT (Eds.) (1996): *Discounting and Intergenerational Equity.* Resources for the Future, Washington DC.

RAMSEY, F. (1928): 'A mathematical theory of savings', *Economic Journal*, **38:** 543–59.

STROTZ, R. H. (1956): 'Myopia and inconsistency in dynamic utility maximization', *Review of Economic Studies*, **23:** 165–180.

UNITED NATIONS THIRD CONFERENCE OF THE PARTIES OF THE FRAMEWORK CONVENTION ON CLIMATE CHANGE (1997): *Kyoto Protocol to the United Nations Framework Convention on Climate Change.* United Nations.

WEITZMAN, M. L. (1998): 'Why the far distant future should be discounted at its lowest possible rate', *Journal of Environmental Economics and Management*, **36:** 201–8.

WEITZMAN, M. L. (2001): 'Gamma discounting', *American Economic Review*, **91:** 260–71.

WINKLER, R. (2002): 'Structural change with joint production of consumption and environmental pollution: A neo-austrian approach'. *Discussion-Paper No. 386*, University of Heidelberg.